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## **GUIDEBOOK**





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## TI-95

# MATHEMATICS

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## GUIDEBOOK

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This manual provides instructions for using the TI-95 Mathematics cartridge. Before using this manual, you should already be familiar with the scientific-calculator functions, described in the *TI-95 User's Guide*.

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# Contents of This Guidebook

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This guidebook describes the programs contained in the TI-95 Mathematics cartridge. The book is organized to help you use the programs.

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## Organization of the Guidebook

The guidebook includes 13 chapters.

Chapter 1 gives a brief description of each program in the cartridge, the general use and care of the cartridge, how to save data for later use, and how to use an optional printer with the programs.

Chapters 2 through 13 provide detailed information about each program in the cartridge. The discussion of each program includes:

- ▶ A brief presentation of general information about the program, the inputs required, and the equations used by the program.
- ▶ Step-by-step instructions for using the program.
- ▶ An example demonstrating the use of the program.

Two appendixes are included at the end of the guidebook.

Appendix A contains a list showing the data registers used with each program. This is useful when storing or retrieving data from one of the programs. Appendix A also includes a list of all flags used in each program.

Appendix B contains service and warranty information that may be useful in case of difficulty.



# Chapter 1: Getting Started

---

This chapter describes the handling, installation, and use of the TI-95 Mathematics cartridge. It also introduces you to the programs available on the cartridge.

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# Installing the Mathematics Cartridge

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You should become familiar with the proper handling and installation of the Mathematics cartridge before using the programs.

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## Handling the Cartridge

Handle the cartridge with the same care you would give any other electronic equipment.

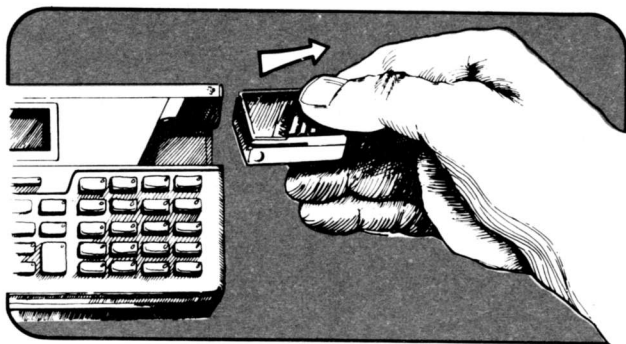
- ▶ Avoid static electricity. Before handling the cartridge, you should touch a metal object to discharge any static electricity.
- ▶ Store the cartridge in its original container or in the cartridge port on the upper right side of the TI-95.

## Installing the Cartridge

The calculator is shipped with a port protector installed in the cartridge port. This protector resembles a cartridge and is installed to prevent dust from accumulating on the electrical contacts inside the port. (It is a good practice to always keep a cartridge or the port protector in the port.)

To install the Mathematics cartridge:

1. **Turn the calculator off.** Installing a cartridge while the TI-95 is on may result in memory loss.
2. If the port protector or another cartridge is already installed, remove it as shown below. Place your thumb on the ridged area at the top of the cartridge and slide it to the right.

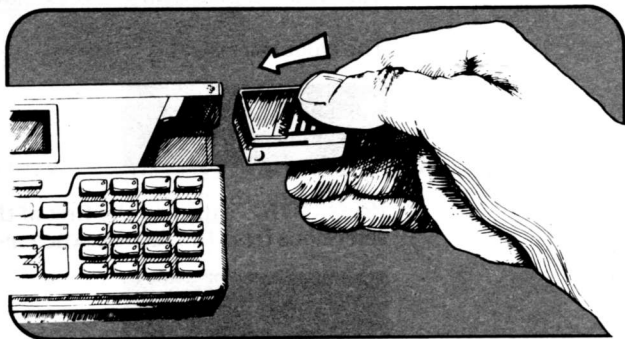


After you remove a cartridge, be sure to store it properly.

---

**Installing  
the Cartridge  
(Continued)**

3. Turn the Mathematics cartridge so that the ridges are facing upward and insert it into the port, small end first.



4. Slide the cartridge to the left until it snaps into place.

# Displaying the Mathematics Menu

---

After installing the cartridge, you can then display the **MATHEMATICS** menu. Each selection on this menu represents a different type of mathematical calculation. For each type of calculation, the cartridge contains either a single program or a "family" of related programs.

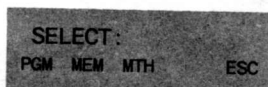
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## Accessing the Mathematics Cartridge

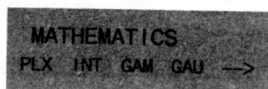
To access the cartridge and display the **MATHEMATICS** menu:

1. Turn the calculator on and press **[RUN]**.

The calculator displays:



2. Press **<MTH>** to display the first of three groups of selections from the **MATHEMATICS** menu.



## The Mathematics Menu

The **MATHEMATICS** menu, which is described on the following pages, enables you to select the type of mathematical calculation you want to perform. Depending on the type you select, the calculator either runs a program or displays another menu.

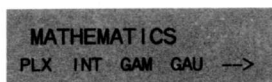
- ▶ If there is only one program that performs that type of calculation, the calculator runs the program.
- ▶ If there are two or more programs that perform the same general type of calculation, the calculator displays a menu that lets you choose the specific program you want to run.



---

**The  
Mathematics  
Menu  
(Continued)**

When you press **RUN** <MTH>, the calculator displays the first group of selections available from the menu.



<PLX>      Runs the Complex Functions program.

<INT>      Displays a menu that lets you select  
            from the following interpolation  
            programs.

        Cubic Splines  
        Exact Polynomials

<GAM>      Runs the Gamma Function program.

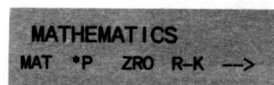
<GAU>      Runs the Gauss Quadrature program.

<-->      Displays the next group of menu  
            selections (shown on the next page).

(continued)

### The Mathematics Menu (Continued)

When you select <--> from the first group of Mathematics selections, the calculator displays the second group.

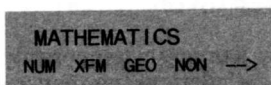


- <MAT> Displays a menu that lets you select from the following matrix algebra programs.
- Matrix Product
  - Inversion/Linear Systems
  - Tridiagonal Systems
  - Eigenvalues
- <\*P> Runs the Polynomial Product program.
- <ZRO> Displays a menu that lets you select from the following root-finding programs.
- The Q-D Method
  - The Bairstow Method
  - The Bisection Method
  - Newton's Method
- <R-K> Runs the Runge-Kutta program.
- <--> Displays the next group of menu selections (shown on the next page).

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**The  
Mathematics  
Menu  
(Continued)**

When you select <--> from the second group of Mathematics selections, the calculator displays the third group.



- |       |   |
|-------|---|
| <NUM> | Runs the Number Theory program.   |
| <XFM> | Runs the Coordinate Transforms program.   |
| <GEO> | Displays a menu that lets you select from the following analytical geometry programs.<br><br>Conic Sections<br>Quadric Surfaces |
| <NON> | Runs the Nonlinear Systems program.   |
| <-->  | Displays the first group of menu selections (shown on page 1-5).  |

# Exiting from a Cartridge Program

---

At any point in a program, you can stop using the program and start using the calculator's built-in functions. You do not need to press any special keys to exit the program. In some instances, however, you may want to return to the same program later or run another program.

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## When You Finish a Program

When you finish using a program, the last menu selections used by the program usually remain in the display. You can clear these selections by pressing the key sequence **2nd** **[F:CLR]**.

## When You Want to Return to a Program

When you leave a program temporarily in order to perform other calculations, you can easily return to the program at the same point from which you left it.

- ▶ If the program's menu selections are still in the display, you can proceed with the program by pressing the applicable key on the menu.
- ▶ If your calculations involved a function such as **CONV**, which displays its own menu, you can return to the previous program menu by pressing the **OLD** key on the calculator.

**Note:** If you leave a program and then run another program that redefines the function keys, you cannot use the **OLD** key to return to the previous program.

## When You Want to Run Another Program

When you want to exit a program and run another one, use one of the methods given below.

- ▶ If you want to exit a program and run another in the same "family," you can use the **<ESC>** key included on the program menus. Press **<ESC>** until the calculator displays the menu that lets you select the other program.
- ▶ If you want to run another program that is not in the same family, press **RUN** **<MTH>** to display the **MATHEMATICS** menu. Then select the new program.

# Using an Optional Printer with the Programs

---

When a printer is connected to the calculator, many of the mathematics programs automatically give you a printed record of calculations. (For information on setting up a printer, refer to "Printer Device Numbers" and "Setting the Printer Format" in Chapter 6 of the *TI-95 User's Guide*.)

---

## Advantages of Using a Printer

A printer gives you a convenient method of reviewing the results of your calculations. The printout includes:

- ▶ The name of the program.
- ▶ The data values you entered. (This also enables you to see if you entered the values correctly.)
- ▶ Any options you selected.
- ▶ The results of the calculations, along with labels that identify each result.

With a printer, the program does not stop to display individual results. (Without a printer, you need to press a key to display each result.) Instead, the program prints a continuous list of results until the output is complete.

## Programs That Do Not Print

The Complex Functions, Gamma Function, and the first three options of the Number Theory program generate only a small amount of output. Because their output is not usually needed for future reference, they do not automatically send information to the printer. To record a displayed number while using one of these programs, press **2nd** **[PRINT]**.

## When to Connect a Printer

When you select a program, the calculator immediately checks to see if a printer is connected.

- ▶ If you want a printout, connect the printer and turn it on before selecting the program.
- ▶ If you do not want a printout, be sure to turn the printer off or disconnect it before selecting the program.

## Precaution When Using a Printer

When you are using a printer with a cartridge program, be sure the calculator is not in the Trace mode. In the Trace mode, the calculator prints each program step as it is executed. This slows the operation of the program and uses a large amount of paper.



# Chapter 2: Performing Complex Functions

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**This chapter describes how to use the Complex Functions program.**

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	<b>Example: Complex Functions .....</b>	<b>2-11</b>

# The Complex Functions Program

---

This program provides mathematical functions that enable the TI-95 to perform calculations on complex numbers.

---

## Introduction

The program consists of an input menu and four categories of mathematical functions.

Although all the functions require complex numbers in rectangular form, the program includes polar/rectangular conversion on the input menu. This feature is useful for converting numbers from polar to rectangular form before performing a calculation, or for converting a result to polar form.

The program's functions operate on two complex numbers called X and Y. All results become the new X number. You can reuse the result of a function in either of two ways.

- ▶ To use the result as the X number in another calculation, you can enter the Y number (if needed) and then select the desired function.
- ▶ To use the result as the Y number in another calculation, you can exchange X and Y from the input menu, enter the new X, and then select the desired function.

The mathematical functions of the program are:

$X + Y$ ,  $X - Y$ ,  $X * Y$ ,  $X / Y$ ,  $1/X$   
 $X^2$ ,  $\sqrt{X}$ ,  $Y^X$ ,  $\sqrt[Y]{Y}$ ,  $\ln(X)$ ,  $e^X$ ,  $\log_Y(X)$   
 $\sin(X)$ ,  $\cos(X)$ ,  $\tan(X)$ , and their inverses  
 $\sinh(X)$ ,  $\cosh(X)$ ,  $\tanh(X)$ , and their inverses

## References

*Handbook of Mathematical Functions*, M. Abramowitz and I. A. Stegun, National Bureau of Standards, 1972.

*Handbook of Engineering Fundamentals*, Ovid W. Eshbach, John Wiley & Sons, Inc., 1954.



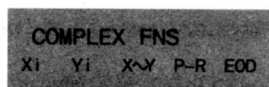
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## Starting the Program

To start the Complex Functions program, select <PLX> from the **MATHEMATICS** menu.

The program selects RAD (radian) angle units. The program's calculations require the RAD mode, so do not change this setting until you stop using the program.

The program displays a menu to let you enter data in either rectangular or polar form.



- |                            |   |
|----------------------------|---|
| <Xi>                       | Enters a pair of values for the X complex number. (You press $\boxed{x \sim t}$ after the first value and this key after the second value.) |
| <Yi>                       | Enters a pair of values for the Y complex number. (You press $\boxed{x \sim t}$ after the first value and this key after the second value.) |
| <X~Y>                      | Exchanges the X and Y complex numbers.  |
| <P~R>                      | Converts X from polar form (in radians) to rectangular form.  |
| $\boxed{\text{INV}}$ <P~R> | Converts X from rectangular to polar form (in radians).   |
| <EOD>                      | Accepts the current values for X and Y, and then proceeds to the next part of the program.  |

### Entering Rectangular Data

To enter the data in rectangular form:

1. Enter the real part of X and press the  $\boxed{x \sim t}$  key to store the value in the t-register.
2. Enter the imaginary part of X and press  $\langle Xi \rangle$ .
3. Enter the real part of Y and press the  $\boxed{x \sim t}$  key to store the value in the t-register.
4. Enter the imaginary part of Y and press  $\langle Yi \rangle$ .
5. Press  $\langle EOD \rangle$  and proceed to page 2-6.

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## Entering Polar Data

Although you can enter the data in polar form, you must convert the values to rectangular form before performing any calculations. Because  $\langle P-R \rangle$  converts only the X values, you use  $\langle X \sim Y \rangle$  in conjunction with  $\langle P-R \rangle$  to convert both variables. The method of entry described here is one of several you may use.

To enter the data in polar form:

1. Enter the magnitude of X and press the  $\boxed{x \sim t}$  key to store the value in the t-register.
2. Enter the angle of X. If it is not in radians, convert it to radians and return to the input menu by pressing  $\boxed{OLD}$ . Then press  $\langle Xi \rangle$ .
3. Press  $\langle P-R \rangle \langle X \sim Y \rangle$  to convert the X values to rectangular form and temporarily store them in the space for the Y values.
4. Enter the magnitude of Y and press the  $\boxed{x \sim t}$  key to store the value in the t-register.
5. Enter the angle of Y. If it is not in radians, convert it to radians and return to the input menu by pressing  $\boxed{OLD}$ . Then press  $\langle Yi \rangle$ .
6. Press  $\langle P-R \rangle \langle X \sim Y \rangle$  to convert the Y values to rectangular form and store both X and Y where they belong.
7. Press  $\langle EOD \rangle$  to indicate you are ready to proceed.

### Types of Functions Available

After you enter the data and press <EOD>, the program displays a menu to let you choose the type of function you want.



ARI PRL CIR HYP INP

- <ARI> Lets you perform arithmetic functions. For instructions, refer to the next page.
- <PRL> Lets you perform power, root, and log functions. For instructions, refer to page 2-8.
- <CIR> Lets you perform circular trigonometric functions. For instructions, refer to page 2-9.
- <HYP> Lets you perform hyperbolic functions. For instructions, refer to page 2-10.
- <INP> Returns to the **COMPLEX FNS** menu to enter (input) new data.

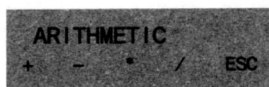
---

## Performing an Arithmetic Function

To perform an arithmetic function on the X and Y data you have entered:

1. Select <ARI> as the type of function.

The program displays:



ARITHMETIC  
+ - \* / ESC

<+> Calculates  $X + Y$ .

<-> Calculates  $X - Y$ .

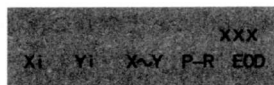
<\*> Calculates  $X * Y$ .

</> Calculates  $X / Y$ .

**INV** </> Calculates  $1/X$ .

2. Press the applicable key for the function you want.

The program displays:



XXX  
X1 Y1 X-Y P-R EDD

where *xxx* is the real part of the result

3. Press the **x~t** key to display the imaginary part of the result.

## Calculating a Power, Root, or Log

To calculate a power, root, or log using the X and Y data you have entered:

1. Select <PRL> as the type of function.

The program displays:

```
POWER/ROOT/LOG
X^2 Y^X ln log ESC
```

<X^2> Calculates  $X^2$ .

**INV** <X^2> Calculates  $\sqrt{X}$ .

<Y^X> Calculates  $Y^X$ .

**INV** <Y^X> Calculates  $\sqrt[Y]{Y}$ .

<ln> Calculates  $\ln X$ .

**INV** <ln> Calculates  $e^X$ .

<log> Calculates  $\log_y X$ .

2. Press the applicable key for the function you want.

The program displays:

```
      XXX
Xi  Yi  X~Y  P-R  EOD
```

where *xxx* is the real part  
of the result

3. Press the **x~t** key to display the imaginary part of the result.

---

Performing  
Circular  
Trigonometric  
Functions

To perform a circular trigonometric function on the X and Y data you have entered:

1. Select <CIR> as the type of function.

The program displays:

```
CIRCULAR FNS
sin cos tan    ESC
```

<sin>      Calculates sin X.

**INV** <sin>      Calculates  $\sin^{-1} X$ .

<cos>      Calculates cos X.

**INV** <cos>      Calculates  $\cos^{-1} X$ .

<tan>      Calculates tan X.

**INV** <tan>      Calculates  $\tan^{-1} X$ .

2. Press the applicable key for the function you want.

The program displays:

```
          xxx
XJ  YI  X~Y  P~R  EOD
```

where xxx is the real part  
of the result

3. Press the **X~t** key to display the imaginary part of the result.

## Performing a Hyperbolic Function

To perform a hyperbolic function on the X and Y data you have entered:

1. Select <HYP> as the type of function.

The program displays:

```
HYPERBOLIC FNS
sh  ch  th      ESC
```

<sh>      Calculates  $\sinh X$ .

**INV** <sh>      Calculates  $\sinh^{-1} X$ .

<ch>      Calculates  $\cosh X$ .

**INV** <ch>      Calculates  $\cosh^{-1} X$ .

<th>      Calculates  $\tanh X$ .

**INV** <th>      Calculates  $\tanh^{-1} X$ .

2. Press the applicable key for the function you want.

The program displays:

```
          xxx
XI  YI  X~Y  P~R  EOD
```

where *xxx* is the real part of the result

3. Press the **INV** key to display the imaginary part of the result.



## Examples: Complex Functions

The examples below demonstrate how to use the Complex Functions program. The first example shows what to do when a calculation operates on more than two complex numbers. The second example includes the entry and conversion of polar numbers.

### Example 1: Rectangular Coordinates

Calculate  $\left(\frac{1}{3-4i} - \sqrt{-5+6i}\right) 2.6i$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <PLX> <b>COMPLEX</b>	
Enter $-5+6i$	5 <b>+/-</b> <b>x↔t</b> 6 <Xi>      Xr=	-5.
Find $\sqrt{-5+6i}$	<EOD> <PRL> <b>INV</b> <x^2>      Xr=	1.185379618
Store the result as Y and enter $3-4i$ for X	<X↔Y> 3 <b>x↔t</b> 4 <b>+/-</b> <Xi>      Xr=	3.
Find $1/(3-4i)$	<EOD> <ARI> <b>INV</b> </>      Xr=	0.12
Use results in subtraction	<EOD> <ARI> <->      Xr=	-1.065379618
Enter $0+2.6i$ as Y so previous result remains	0 <b>x↔t</b> 2.6 <Yi>      Yr=	0.
Multiply numbers and display the real part	<EOD> <ARI> <*>      Xr=	6.164170507
Display imaginary part	<b>x↔t</b>	-2.769987006

The result is  $6.164170507 + i(-2.769987006)$ .

## Examples: Complex Functions (Continued)

### Example 2: Polar Coordinates

Calculate  $\sin(100 \angle 30^\circ \div 40 \angle -90^\circ)$  and show the result in polar form.

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <PLX>	COMPLEX
Convert $40 \angle -90^\circ$ to radians	40 <b>x~t</b> 90 <b>+/-</b> <b>CONV</b> <ANG> <D-R>	Rad = -1.570796327
Convert to rectangular and store result as Y	<b>OLD</b> <Xi> <P-R> <X~Y>	Xr = 0.
Convert $100 \angle 30^\circ$ to radians	100 <b>x~t</b> 30 <b>CONV</b> <ANG> <D-R>	Rad = .5235987756
Convert to rectangular	<b>OLD</b> <Xi> <P-R>	Xr = 86.60254038
Divide	<EOD> <ARI> </>	Xr = -1.25
Calculate the sine	<EOD> <CIR> <sin>	Xr = -4.189718716
Convert to polar and show magnitude	<b>INV</b> <P-R>	r = 4.403674223
Show angle (in radians)	<b>x~t</b>	2.828593336
Convert to degrees	<b>CONV</b> <ANG> <b>INV</b> <D-R>	Deg = 162.0664601

## Chapter 3: Interpolating Points on a Curve

---

This chapter shows you how to use the Cubic Splines and Exact Polynomials programs to interpolate points on a curve.

---

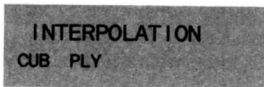
<b>Table of Contents</b>	<b>Introduction</b> .....	<b>3-2</b>
	<b>The Cubic Splines Program</b> .....	<b>3-3</b>
	<b>Cubic Splines with Known Data Points</b> .....	<b>3-4</b>
	<b>Example: Cubic Splines with Known Data Points</b> .....	<b>3-10</b>
	<b>Cubic Splines with Known Derivatives</b> .....	<b>3-12</b>
	<b>Example: Cubic Splines with Known Derivatives</b> .....	<b>3-16</b>
	<b>The Exact Polynomials Program</b> .....	<b>3-18</b>
	<b>Exact Polynomials with Known Data Points</b> .....	<b>3-19</b>
	<b>Example: Exact Polynomials with Known Data Points</b> ..	<b>3-24</b>
	<b>Exact Polynomials with Known Coefficients</b> .....	<b>3-26</b>
	<b>Example: Exact Polynomials with Known Coefficients</b> ..	<b>3-30</b>

Both of the interpolation programs in this chapter—Cubic Splines and Exact Polynomials—can be used in either of two ways, depending on the type of data you have.

---

## The Interpolation Programs

When you select <INT> from the **MATHEMATICS** menu, the calculator displays a menu that lets you select either the Cubic Splines or the Exact Polynomials program.



INTERPOLATION  
CUB PLY

The Cubic Splines program can be used with:

- ▶ Known data points. (Data points are considered “new” data.)
- ▶ Known second derivatives. (Derivatives are considered “old” data.)

The Exact Polynomials program can be used with:

- ▶ Known data points. (Data points are considered “new” data.)
- ▶ Known coefficients. (Coefficients are considered “old” data.)

# The Cubic Splines Program

---

This program calculates a sequence of cubic polynomials that form a continuous curve intersecting all the points you enter.

---

## Introduction

The Cubic Splines program enables you to interpolate between known points more accurately than linear interpolation when the known points do not lie along a straight line. The technique used by cubic splines assumes the points lie along a curve whose second derivative is continuous over the entire interval of known data.

The number of points you enter is designated as  $m$ . You must partition for at least  $6m + 4$  registers before running the program, and the program requires at least three data points.

The program fits cubic polynomials to successive points in the order the points are entered. Therefore, it is important to enter the points in the order that they occur along the curve.

The program lets you specify the derivatives of the endpoints,  $f'(X_1)$  and  $f'(X_m)$ , in either of two ways.

- ▶ You can let the program use second derivatives of zero at the endpoints.
- ▶ You can enter the actual second derivatives at the endpoints for a more accurate fit.

## Reference

*Numerical Methods*, Robert W. Hornbeck, Quantum Publishers, Inc., 1975, pp. 47-50.

# Cubic Splines with Known Data Points

---

If you know only the data points, use the following procedure to run the Cubic Splines program.

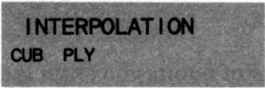
---

## Starting the Program

To start the Cubic Splines program:

1. Select <INT> from the **MATHEMATICS** menu.

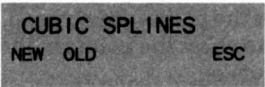
The program displays:



```
INTERPOLATION
CUB  PLY
```

2. Press <CUB>.

The program displays:



```
CUBIC SPLINES
NEW  OLD          ESC
```

3. Press <NEW>.

You can then begin entering the data points.

## Entering the Data Points

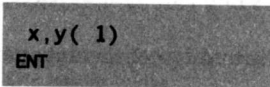
When you select <NEW> from the **CUBIC SPLINES** menu, the program displays:



m      EOD      ESC

1. Enter the number of data points and press <m>.
2. Press <EOD>.

The program prompts you to enter the x and y values for each data point, beginning with the leftmost point.



x,y( 1)  
ENT

3. Enter the x and y values as described below.
  - ▶ Enter the x value and press the **[x↔t]** key to store it in the t-register.
  - ▶ Enter the y value and press <ENT>.

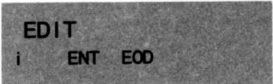
If you need to remove the last x and y values that were entered with <ENT>, press **[INV]** <ENT>.

4. Repeat step 3 until all of the x and y values are entered.

The program then displays the menu shown on the following page.

### The Edit Menu

After you enter the x and y values for all the data points, the program displays an **EDIT** menu that allows you to change any of the values.



```
EDIT
i  ENT  EOD
```

- ▶ If you do not want to edit any values, press <EOD> and go to the next page.
- ▶ If you want to edit a value, use the following procedure.

### Editing Data Points

To edit the x or y value for a specified data point:

1. Enter the number of the data point and press <i>.

The program displays the x value and puts the y value in the t-register.

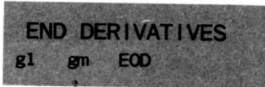
2. Use the  $\boxed{x \leftrightarrow t}$  key and the data entry keys to store the correct x value in the t-register and display the correct y value.
3. Press <ENT> to enter the correct values.
4. Repeat steps 1 through 3 for any other points you want to edit.
5. Press <EOD> to continue with the program.



---

## Entering the End Derivatives

After you enter all the data points and select <EOD> from the **EDIT** menu, the program prompts you to specify the second derivatives at the endpoints.



```
END DERIVATIVES
g1  gm  EOD
```

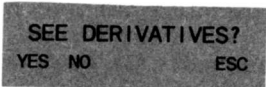
The program normally assumes that the end derivatives are zero. You can accept this value by pressing <EOD>.

However, if you want to specify other values:

1. Enter the second derivative for the leftmost point and press <g1>.
2. Enter the second derivative for the rightmost point and press <gm>.
3. Press <EOD> to continue with the program.

## Choosing the Results

After you press <EOD>, the program displays:



```
SEE DERIVATIVES?
YES NO          ESC
```

- ▶ To display the second derivative at each point, press <YES> and go to "Viewing the Derivatives" on the next page.
- ▶ To interpolate, press <NO> and go to "Interpolating" on page 3-9.
- ▶ To restart the Cubic Splines program and enter different data, press <ESC>.

### Viewing the Derivatives

When you select <YES> from the **SEE DERIVATIVES?** menu, the program displays the second derivative at the first (leftmost) data point.

g 1=  
NXT

xxx

where xxx is the value of g1

Press <NXT> repeatedly to display the second derivative at each of the remaining points.

When the last derivative is displayed, pressing <NXT> redisplay the **SEE DERIVATIVES?** menu. Selecting <NO> from that menu lets you interpolate.

## Interpolating

When you select <NO> from the **SEE DERIVATIVES?** menu, the program displays:

```
INTERPOLATE?  
YES NO
```

1. Press <YES>.

The program displays:

```
x          0.  
          ESC
```

2. Enter a value for x and press <x>.

The program displays the corresponding value of y.

```
y=  
x          xxx
```

 where xxx is the value of y

3. Repeat step 2 for any other values of x.
4. Press <ESC> to return to the **SEE DERIVATIVES?** menu.

## Example: Cubic Splines with Known Data Points

The area below a given ordinate of the normal curve does not vary linearly. Because a table describing the normal curve states the area at fixed increments, you can use the Cubic Splines program to interpolate the area accurately.

**Example** A table of areas for the normal curve is listed below. Find the area for  $z = .8$ .

$z$	.6	.7	.8	.9	1.0	1.1
$P(z)$	.725747	.758036	---	.815940	.841345	.864334
$P'(z)$	-.1999347					-.2396373

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <INT> <CUB> <NEW>	CUBIC SPLINES
Enter the number of points	5 <M> <EOD>	x,y( 1)
Enter the first point	.6 <b>x↔t</b> .725747 <ENT>	x,y( 2)
Enter the second point	.7 <b>x↔t</b> .758036 <ENT>	x,y( 3)
Enter the third point	.9 <b>x↔t</b> .815940 <ENT>	x,y( 4)
Enter the fourth point	1 <b>x↔t</b> .841345 <ENT>	x,y( 5)
Enter the fifth point	1.1 <b>x↔t</b> .864334 <ENT>	EDIT
Proceed with program	<EOD>	END DERIVATIVES
Enter second derivatives at the endpoints	.1999347 <b>+/-</b> <g1> .2396373 <b>+/-</b> <gm> <EOD>	SEE DERIVATIVES?

**Example  
(Continued)**

Procedure	Press	Display
View the derivatives	<YES>	g 1 = -0.1999347
	<NXT>	g 2 = - .2200526828
	<NXT>	g 3 = - .2409746016
	<NXT>	g 4 = - .2422470246
	<NXT>	g 5 = - .2396373
	<NXT>	SEE DERIVATIVES?
Proceed with program	<NO>	INTERPOLATE?
Interpolate at .8	<YES> .8	
	<x>	y = .7881405682

The mathematics of the normal curve calculate the actual value as .78814467. Compared to this, the result of .7881405682 has an error of .00052%.

The most commonly used interpolation is linear. By linear interpolation  $((.8 - .7)/(.9 - .7) \times (.815940 - .758036) + .758036)$ , you get a less accurate answer (.786988). Compared to the actual value, this result has an error of .147%.

The error is calculated from the equation

$$\% \text{error} = \left( \frac{\text{Observed value}}{\text{Actual value}} - 1 \right) \times 100\%$$

## Cubic Splines with Known Derivatives

---

If you know the second derivative at each of the data points, use the following procedure to run the Cubic Splines program.

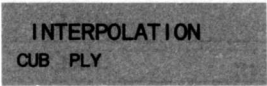
---

### Starting the Program

To start the Cubic Splines program:

1. Select <INT> from the **MATHEMATICS** menu.

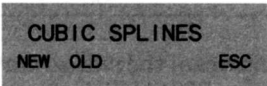
The program displays:



```
INTERPOLATION
CUB  PLY
```

2. Press <CUB>.

The program displays:



```
CUBIC SPLINES
NEW  OLD      ESC
```

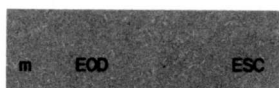
3. Press <OLD>.

You can then begin entering the second derivative at each data point.

---

## Entering the Derivatives

When you select <OLD> from the **CUBIC SPLINES** menu, the program displays:



```
m      EOD      ESC
```

1. Enter the number of data points and press <m>.
2. Press <EOD>.

The program prompts you to enter the second derivative at each data point, beginning with the leftmost point.



```
g( 1)  
ENT
```

3. Enter the derivative and press <ENT>.
4. Repeat step 3 until all the derivatives are entered.

The program then displays the menu shown on the following page.

### The Edit Menu

After you enter all the second derivatives, the program displays an **EDIT?** menu that allows you to change any of the values.

```
EDIT?  
YES NO
```

- ▶ If you do not want to edit any values, press <NO> and go to the next page.
- ▶ If you want to edit a value, press <YES> and use the procedure below.

### Editing the Derivatives

When you select <YES> from the **EDIT?** menu, the program displays the same menu that you used to enter the second derivatives.

```
g( 1)  
ENT
```

- ▶ To display the first derivative, press the **CE** key.
- ▶ To accept the current value and proceed to the next derivative, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you press <ENT> to accept or edit the value of the last (rightmost) derivative, the program returns to the **EDIT?** menu. Selecting <NO> from that menu lets you interpolate.



## Interpolating

When you select <NO> from the **EDIT?** menu, the program displays:

```

                                0.
x                               ESC
```

To interpolate:

1. Enter a value for x and press <x>.

The program displays the corresponding value of y.

```

y=          xxx
x           ESC
```

where xxx is the value of y

2. Repeat step 1 for any other values of x.

## Exiting the Program

When you are finished interpolating and are ready to exit the program:

1. Press <ESC>.

The program displays the following menu. (This menu is normally used for "Cubic Splines with Known Data Points," as described on page 3-7.)

```

SEE DERIVATIVES?
YES NO          ESC
```

2. Press <ESC> to return to the **CUBIC SPLINES** menu.

## Example: Cubic Splines with Known Derivatives

The example on page 3-10 produced the second derivatives at each entered point. The following example uses those derivatives to arrive at the same result.

**Example** From the second derivatives listed below, interpolate the function at .8.

$x$	$f''(x_i)$
.6	-.1999347
.7	-.2200526828
.9	-.2409746016
1.0	-.2422470246
1.1	-.2396373

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <INT> <CUB> <OLD>	CUBIC SPLINES
Enter the number of points	5 <m> <EOD>	g(1)
Enter second derivatives	.1999347 <b>+/-</b> <ENT>	g(2)
	.2200526828 <b>+/-</b> <ENT>	g(3)
	.2409746016 <b>+/-</b> <ENT>	g(4)
	.2422470246 <b>+/-</b> <ENT>	g(5)
	.2396373 <b>+/-</b> <ENT>	EDIT?
Interpolate at .8	<NO> .8 <x>	y= .7881405682
Repeat the problem	<ESC> <ESC> <OLD>	CUBIC SPLINES

**Example**  
**(Continued)**

Recalculate the interpolation using zero for the endpoint second derivatives.

Procedure	Press	Display
Enter the number of points	5 <m> <EOD>	g(1)
Enter second derivatives	0 <ENT>	g(2)
	<b>CE</b> <ENT>	g(3)
	<b>CE</b> <ENT>	g(4)
	<b>CE</b> <ENT>	g(5)
	0 <ENT>	EDIT?
Interpolate at .8	<NO> .8 <x>	.7881405682

The result is the same as for nonzero endpoint second derivatives. The endpoints are too distant from the point of interpolation for a small change in the endpoint second derivatives to have any effect on the result.

# The Exact Polynomials Program

---

This program generates a polynomial that intersects all the entered points. You can then use the program to determine y values for additional x values.

---

## Introduction

The Exact Polynomials program uses the Divided Differences method to compute a polynomial that fits all the entered data points.

You must partition the calculator for at least 74 data registers before running the program.

The program can accept up to 21 data points. The polynomial generated has the lowest order that can intersect all the points, with a maximum order of 20.

## Reference

*Handbook of Engineering Fundamentals*, Ovid W. Eshbach, John Wiley & Sons, Inc., 1954.

## Exact Polynomials with Known Data Points

---

If you know only the data points, use the following procedure to run the Exact Polynomials program.

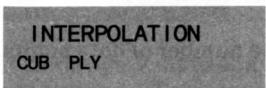
---

### Starting the Program

To start the Exact Polynomials program:

1. Select <INT> from the **MATHEMATICS** menu.

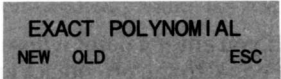
The program displays:



INTERPOLATION  
CUB PLY

2. Press <PLY>.

The program displays:



EXACT POLYNOMIAL  
NEW OLD ESC

3. Press <NEW>.

You can then begin entering the data points.

### Entering the Data Points

When you select <NEW> from the **EXACT POLYNOMIAL** menu, the program displays:

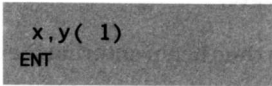


n      EOD      ESC

1. Enter the number of data points and press <n>.
2. Press <EOD>.

The program prompts you to enter the x and y values for each data point, beginning with the leftmost point.

**Note:** If you enter a point whose x-coordinate is less than or equal to the previous x entry, the program rejects the entry.



x,y( 1)  
ENT

3. Enter the x and y values as described below.
  - ▶ Enter the x value and press the  $\boxed{x \rightarrow t}$  key to store it in the t-register.
  - ▶ Enter the y value and press <ENT>.

If you need to remove the last x and y values that were entered with <ENT>, press  $\boxed{INV}$  <ENT>.

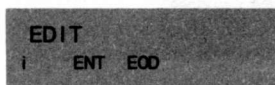
4. Repeat step 3 until all of the x and y values are entered.

The program then displays the menu shown on the following page.

---

## The Edit Menu

After you enter the x and y values for all the data points, the program displays an **EDIT** menu that allows you to change any of the values.



- ▶ If you do not want to edit any values, press  $\langle \text{EOD} \rangle$  and go to the next page.
- ▶ If you do want to edit a value, use the following procedure.

## Editing Data Points

To edit the x or y value for a specified data point:

1. Enter the number of the data point and press  $\langle \text{I} \rangle$ .

The program displays the x value and puts the y value in the t-register.

2. Use the  $\boxed{x \sim t}$  key and the data entry keys to store the correct x value in the t-register and display the correct y value.
3. Press  $\langle \text{ENT} \rangle$  to enter the correct values.
4. Repeat steps 1 through 3 for any other points you want to edit.
5. Press  $\langle \text{EOD} \rangle$  to continue with the program.

### Choosing the Results that Are Displayed

When you select <EOD> from the **EDIT** menu, the program displays:

```
SEE COEFF?  
YES NO      ESC
```

- ▶ To display the coefficients of the resulting polynomial, press <YES> and go to "Viewing the Coefficients" below.
- ▶ To interpolate, press <NO> and go to "Interpolating" on the next page.
- ▶ To restart the Exact Polynomials program and enter different data, press <ESC>.

### Viewing the Coefficients

When you select <YES> from the **SEE COEFF?** menu, the program displays the coefficient of the  $x^0$  term in the resulting polynomial.

```
c 0=      xxx  
NXT
```

where *xxx* is the value of  $c_0$

Press <NXT> repeatedly to display the coefficient of each of the remaining terms.

When the last coefficient is displayed, pressing <NXT> redisplay the **SEE COEFF?** menu. Selecting <NO> from that menu lets you interpolate.



## Interpolating

When you select <NO> from the **SEE COEFF?** menu, the program displays:

```
INTERPOLATE?  
YES NO
```

1. Press <YES>.

The program displays:

```
                                0.  
x                                ESC
```

2. Enter a value for x and press <x>.

The program displays the corresponding value of y.

```
y=          xxx      where xxx is the value of  
x          ESC      y
```

3. Repeat step 2 for any other values of x.
4. Press <ESC> to return to the **SEE COEFF?** menu.

## Example: Exact Polynomials with Known Data Points

When you need to describe a set of points in terms of a function, you can use the Exact Polynomials program to determine a polynomial approximation.

**Example** Find a polynomial that fits the given numbers and interpolate for  $x = 3.1$ .

x	y
0	6
1	7
2	14
3	33
4	70

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <INT> <PLY> <NEW>	EXACT POLYNOMIAL
Enter the number of points	5 <n> <EOD>	x,y( 1)
Enter the points	0 <b>x↔t</b> 6 <ENT>	x,y( 2)
	1 <b>x↔t</b> 7 <ENT>	x,y( 3)
	2 <b>x↔t</b> 14 <ENT>	x,y( 4)
	3 <b>x↔t</b> 33 <ENT>	x,y( 5)
	4 <b>x↔t</b> 70 <ENT>	EDIT

Example  
(Continued)

Procedure	Press	Display
Proceed with program	<EOD>	SEE COEFF?
View the coefficients	<YES>	c 0 = 6.
	<NXT>	c 1 = 0.
	<NXT>	c 2 = 0.
	<NXT>	c 3 = 1.
	<NXT>	SEE COEFF?
Proceed with program	<NO>	INTERPOLATE?
Interpolate at 3.1	<YES> 3.1	
	<x>	y = 35.791

The polynomial is  $y = 6 + 0x + 0x^2 + 1x^3$ , which can be written as  $y = 6 + x^3$ .

## Exact Polynomials with Known Coefficients

---

If you know the coefficients of the polynomial, use the following procedure to run the Exact Polynomials program.

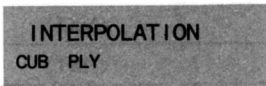
---

### Starting the Program

To start the Exact Polynomials program:

1. Select <INT> from the **MATHEMATICS** menu.

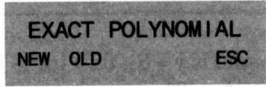
The program displays:



```
INTERPOLATION
CUB PLY
```

2. Press <PLY>.

The program displays:



```
EXACT POLYNOMIAL
NEW OLD ESC
```

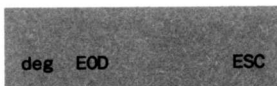
3. Press <OLD>.

You can then begin entering the coefficients of the polynomial.

---

## Entering the Coefficients

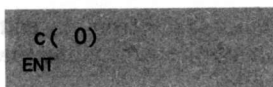
When you select <OLD> from the **EXACT POLYNOMIAL** menu, the program displays:



deg EOD ESC

1. Enter the degree of the polynomial and press <deg>.
2. Press <EOD>.

The program prompts you to enter the coefficient of each term, beginning with the  $x^0$  term.



c ( 0)  
ENT

3. Enter the coefficient and press <ENT>.
4. Repeat step 3 until all the coefficients are entered.

The program then displays the menu shown on the following page.

### The Edit Menu

After you enter all the coefficients, the program displays an **EDIT?** menu that allows you to change any of the values.

```
EDIT?  
YES NO
```

- ▶ If you do not want to edit any values, press **<NO>** and go to the next page.
- ▶ If you want to edit a value, press **<YES>** and use the procedure below.

### Editing the Coefficients

When you select **<YES>** from the **EDIT?** menu, the program displays the same menu that you used to enter the coefficients.

```
c( 0)  
ENT
```

- ▶ To display the current value of the first coefficient, press the **[CE]** key.
- ▶ To accept the current value and proceed to the next coefficient, press **<ENT>**.
- ▶ To edit the current value, enter the correct value and press **<ENT>**.

When you press **<ENT>** to accept or edit the value of the last coefficient, the program returns to the **EDIT?** menu. Selecting **<NO>** from that menu lets you interpolate.

## Interpolating

When you select <NO> from the **EDIT?** menu, the program displays:

```
0.  
ESC  
x
```

To interpolate:

1. Enter a value for x and press <x>.

The program displays the corresponding value of y.

```
y=  
x      xxx  
      ESC
```

where xxx is the value of y

2. Repeat step 1 for any other values of x.

## Exiting the Program

When you are finished interpolating and are ready to exit the program:

1. Press <ESC>.

The program displays the following menu. (This menu is normally used for "Exact Polynomials with Known Data Points," as described on page 3-22.)

```
SEE COEFF?  
YES NO      ESC
```

2. Press <ESC> to return to the **EXACT POLYNOMIAL** menu.

## Example: Exact Polynomials with Known Coefficients

When you need to find points from a known polynomial, you can enter the coefficient of each term into the Exact Polynomials program and find a y value for any x value.

### Example

Find the values of y when x is 1, 2, 2.5, and 4 for the polynomial  $y = 1 + 2x + 3x^2 + x^3$ .

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <INT> <PLY> <OLD>	EXACT POLYNOMIAL
Enter the degree	3 <deg> <EOD>	$\alpha(0)$
Enter the coefficients	1 <ENT>	$\alpha(1)$
	2 <ENT>	$\alpha(2)$
	3 <ENT>	$\alpha(3)$
	1 <ENT>	EDIT?
Proceed with program	<NO>	0.
Enter each x value	1 <x>	y = 7.
	2 <x>	y = 25.
	2.5 <x>	y = 40.375
	4 <x>	y = 121.



This chapter describes how to use the Gamma Function program.

Table of Contents	The Gamma Function Program	4-2
	Example: The Gamma Function	4-4

# The Gamma Function Program

---

This program lets you calculate the gamma function of a number and the natural log of gamma. The program is designed to recognize out-of-range results and to compute  $\ln|\text{gamma}|$  instead of gamma when gamma is too large.

---

## Introduction

The gamma function is defined by the improper integral

$$G = \text{gamma}(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$

which converges for noninteger values of  $x < 1$  and all values of  $x > 0$ . The gamma function is related to the factorial function:  $\text{gamma}(x+1) = x!$ .

The approximate input range of gamma is

$$-70.064 \leq x \leq 70.957$$

where  $x$  is not an integer less than 1.

Although the calculator cannot display gamma for results whose exponent exceeds 99, you can still determine the magnitude of gamma from the  $\ln|\text{gamma}|$  using the laws of logarithms. The mantissa of gamma is

$$\text{Antilog}(\text{Frac}(\ln G / \ln 10))$$

and the exponent of scientific notation for gamma is

$$\text{Intg}(\ln G / \ln 10).$$

## Reference

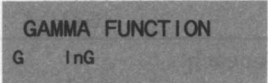
*Handbook of Mathematical Functions*, M. Abramowitz and I. A. Stegun, National Bureau of Standards, 1972.

## Using the Program

To use the Gamma Function program:

1. Select <GAM> from the **MATHEMATICS** menu.

The calculator displays the **GAMMA FUNCTION** menu.




GAMMA FUNCTION  
G    lnG

<G> Calculates the gamma function of the displayed value or the natural log of gamma if the result is too large.

<lnG> Calculates the natural log of the gamma function of the displayed value.

2. Enter the value and press the applicable key.

The program displays the result. For example, if you press <G>, the following is displayed.



G=                      xxx    where xxx is the result  
G    lnG

3. Repeat step 2 for other values.

# Example: The Gamma Function

The example below demonstrates how to use the gamma function.

**Example** Find the gamma and the  $\ln|\text{gamma}|$  of .5, -44.9, and 71.

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <GAM>	GAMMA FUNCTION
Calculate for each value	.5 <G>	G = 1.772453851
	.5 <lnG>	lnG = .5723649429
	44.9 <b>+/-</b> <G>	G = -1.24483 - 55
	44.9 <b>+/-</b> <lnG>	lnG = -126.4231815
	71 <G>	lnG = 230.4390436

You can use the  $\ln|\text{gamma}|$  to calculate gamma. The mantissa of gamma of 71 is

$$\text{Antilog}(\text{Frac}(230.4390436/\ln 10))$$

and the exponent of scientific notation for gamma is

$$\text{Intg}(230.4390436/\ln 10).$$

You can use this method to show that the gamma of 71 is  $1.197857167 \times 10^{100}$ .

## Chapter 5: Integrating a Function

---

This chapter shows you how to use the Gauss Quadrature program to integrate a function.

---

Table of Contents	The Gauss Quadrature Program .....	5-2
	Example: Gauss Quadrature .....	5-5

# The Gauss Quadrature Program

---

This program computes the definite integral of a function you have defined in program memory.

---

## Introduction

The Gauss Quadrature method of integration used by the program incorporates a seven-point Gaussian integration formula for an arbitrary interval.

You must store the function as a subroutine in program memory before running the Gauss Quadrature program. When the program calls the subroutine, the value for the independent variable is available in both the display and in data register H (007).

The number of subintervals affects the computation time and the accuracy. Fewer subintervals allow a quick result. More subintervals minimize error. Consider both factors when deciding on a number of subintervals.

## Reference

*Handbook of Mathematical Functions*, M. Abramowitz and I. A. Stegun, National Bureau of Standards, 1972, p. 887.

**For the  
Beginning  
Programmer**

Before running the Gauss Quadrature program, you must store the function you are integrating in program memory as a subroutine.

If you are not familiar with keystroke programming, refer to the following chapters in the *TI-95 Programming Guide* for instructions.

- ▶ “Working with Programs on the TI-95”
- ▶ “Using Calculator Keystrokes in a Program”
- ▶ “Controlling the Sequence of Operations”

**Rules for  
Storing  
the Function**

When you store the function as a subroutine, follow these rules.

- ▶ The subroutine must be labeled  $fx$  (lowercase only).
- ▶ Anytime the subroutine needs the independent variable, use a RCL H or RCL 007 instruction.
- ▶ The subroutine must terminate with a RTN instruction.

## The Gauss Quadrature Program (Continued)

### Running the Program

To run the Gauss Quadrature program:

1. Store the function in program memory, as described on the previous page.
2. If your subroutine involves a trig function, select the angle units you want to use.
3. Select <GAU> from the **MATHEMATICS** menu.

The program displays:

```
GAUSS QUADRATURE
n  LO  HI  EOD
```

4. Enter the number of subintervals and press <n>.
5. Enter the lower limit and press <LO>.
6. Enter the upper limit and press <HI>.
7. Press <EOD> to evaluate the integral.

While the program is calculating the result, "F4:EOD" is displayed. Depending on the function you are integrating, it may take several minutes to evaluate the integral.

When the calculation is complete, the result is displayed.

```
I =          xxx
          ESC
```

where xxx is the result

8. Press <ESC> to return to the **GAUSS QUADRATURE** menu.



## Example: Gauss Quadrature

The example below demonstrates how to use the Gauss Quadrature program.

Example

Evaluate the integral below using four subintervals.

$$\int_0^2 \frac{e^{2x}}{(x+2)^2} dx$$

Procedure	Press	Display
Enter the function into program memory	<b>LEARN</b> <1st> <b>2nd</b> <b>[LBL]</b> <b>2nd</b> <b>f</b> <b>2nd</b> <b>x</b> <b>(</b> <b>(</b> <b>RCL</b> <b>H</b> <b>*</b> <b>2</b> <b>)</b> <b>INV</b> <b>LN</b> <b>/</b> <b>(</b> <b>RCL</b> <b>H</b> <b>+</b> <b>2</b> <b>)</b> <b>x<sup>2</sup></b> <b>)</b> <b>2nd</b> <b>[RTN]</b> <b>LEARN</b>	
Select the program	<b>RUN</b> <MTH> <GAU>	GAUSS QUADRATURE
Enter the number of subintervals	4 <n>	n = 4.
Enter the lower limit	0 <LO>	LO = 0.
Enter the upper limit	2 <HI>	HI = 2.
Proceed with result	<EOD>	I = 2.263036903



# Chapter 6: Solving Matrices

This chapter describes how to use the programs available through the <MAT> (Matrix Algebra) selection of the MATHEMATICS menu.

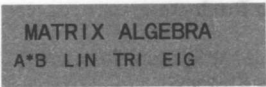
Table of Contents	Introduction .....	6-2
	The Matrix Product Program .....	6-3
	Example: Matrix Products .....	6-9
	The Inversion/Linear Systems Program .....	6-12
	Example: Linear Systems .....	6-21
	The Tridiagonal Systems Program .....	6-23
	Example: Tridiagonal Systems .....	6-28
	The Eigenvalues Program .....	6-30
	Example: Eigenvalues .....	6-34

Advanced Engineering Mathematics, C. R. Wylie, Jr.  
McGraw-Hill Book Company, 1968

Four programs are available from the MATRIX ALGEBRA menu.

## The Matrix Algebra Programs

When you select <MAT> from the **MATHEMATICS** menu, the calculator displays:



MATRIX ALGEBRA  
A\*B LIN TRI EIG

- |   |   |
|---|---|
| <p>&lt;A*B&gt;</p> <p>&lt;LIN&gt;</p> <p>&lt;TRI&gt;</p> <p>&lt;EIG&gt;</p> | <p>Runs the Matrix Product program, described on the next page.</p> <p>Runs the Inversion/Linear Systems program, described on page 6-12.</p> <p>Runs the Tridiagonal Systems program, described on page 6-23.</p> <p>Runs the Eigenvalues program, described on page 6-30.</p> |
|---|---|

# The Matrix Product Program

---

This program multiplies two matrices. You can save one of the matrices for future use.

---

## Introduction

The Matrix Product program multiplies matrix A by matrix B to result in matrix C. The size of matrix A is  $M \times N$ , the size of matrix B is  $N \times P$ , and the size of matrix C is  $M \times P$ .

Before running the program, you must partition the calculator for at least  $10 + (M + 1)(N + 1)$  data registers.

After you enter the complete matrix A, the program prompts you for the first column of matrix B. When you have entered that column, the program generates the first column of matrix C. Then you enter the second column of matrix B and obtain the second column of matrix C. The column-by-column pattern continues until the last column of matrix B is multiplied with matrix A.

You can reuse matrix A for additional multiplications. However, matrix C is not available for further multiplication unless you enter each element as a new matrix A.

You can save the A matrix as a data file anytime after entering (and, if necessary, editing) the matrix. To save the A matrix, you must specify  $10 + nm$  data registers, starting with register 000. For information on saving data as a file, refer to the *TI-95 Programming Guide*.

## Reference

*Advanced Engineering Mathematics*, C. R. Wylie, Jr., McGraw-Hill Book Company, 1966.

## Before Starting the Program

If you have previously saved the A matrix as a file and want to use that data, you must load the data into the data registers before running the Matrix Product program. For information on loading a data file, refer to the *TI-95 Programming Guide*.

## Starting the Program

To start the Matrix Product program:

1. Select <MAT> from the **MATHEMATICS** menu.

The program displays the **MATRIX ALGEBRA** menu.

2. Press <A\*B>.

The program displays:

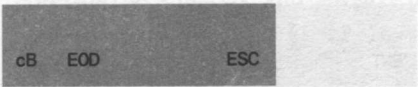
```
MATRIX PRODUCT
NEW OLD      ESC
```

3. Select the option that applies to your data.

- ▶ To enter a new matrix, press <NEW> and proceed to "If Entering New Data" on the next page.
- ▶ To use data you have loaded from the file space or from a storage device, press <OLD> and proceed to "If Using Old Data" on the next page.

**Using Old Data**

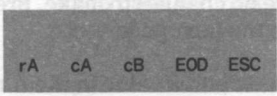
If you select <OLD> from the **MATRIX PRODUCT** menu, the program displays:



1. Enter the number of columns in the B matrix and press <cB>.
2. Press <EOD>.
3. Proceed to "Entering a Column in the B Matrix" on page 6-7.

**Entering New Data**

If you are specifying new values for the A matrix, the program displays:



1. Enter the number of rows in the A matrix and press <rA>.
2. Enter the number of columns in the A matrix and press <cA>.
3. Enter the number of columns in the B matrix and press <cB>.
4. Press <EOD> to indicate you have entered the data.

### Entering the A Matrix

When you press <EOD>, the program prompts you for the values in the A matrix, one column at a time.

```
a( 1, 1)
ENT
```

Enter all the values for the A matrix, pressing <ENT> after each value.

### The Edit Menu

After you have entered all the values, the program displays:

```
EDIT
[~] ENT EOD
```

- ▶ If you have no changes to make, press <EOD> and proceed to the next page.
- ▶ If you want to edit a value in the A matrix, use the procedure described below.

### Editing a Value in the A Matrix

To edit a value:

1. Enter the row number of the value and press the x~t key to store the number in the t-register.
2. Enter the column number of the value and press <[~]>.

The program displays the current value.

3. Enter the new value and press <ENT>. (If the displayed value is already correct, press <ENT>.)
4. Press <EOD> to leave the **EDIT** menu.



### Entering a Column in the B Matrix

After you enter the A matrix, the program prompts you to enter the first column of the B matrix. (Anytime after entering the A matrix, you can save it as a data file.)

```
b( 1, 1)
ENT
```

Enter all the values for the indicated column of the B matrix, pressing <ENT> after each value.

### The Edit Menu

After you have entered a column, the program displays:

```
EDIT?
YES NO
```

- ▶ If you have no changes to make, press <NO> and proceed to the next page.
- ▶ If you want to edit a value, press <YES> and use the procedure described below.

### Editing the Column Entries

When you select <YES> from the **EDIT?** menu, the program displays the same menu that you used to enter the column values.

- ▶ To display the current value, press the **CE** key.
- ▶ To accept the current value and proceed to the next derivative, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you have finished editing the column, select <NO> from the **EDIT?** menu.

### Displaying a Column in the Resulting Matrix

When you select <NO> from the **EDIT?** menu, the program displays a message indicating the column of the resulting matrix that you can examine.



```
col= 1
NXT
```

To examine the results in the indicated column:

1. Press <NXT>.

The program displays the value in the first row of the column.



```
c 1= xxx
NXT
```

 where *xxx* is the value

2. Press <NXT> to display the remaining values in the current column.

When you have displayed all the values in the column, the program proceeds in one of two ways.

- ▶ If there are more columns in the B matrix, the program lets you enter values for the next column.
- ▶ If there are no more columns in the B matrix, the program returns to the **MATRIX PRODUCT** menu.

## Example: Matrix Products

The example below demonstrates how to use the Matrix Products program.

**Example**

Multiply the following matrices.

$$\begin{vmatrix} 12 & 3 & 4 \\ 9 & 10 & 7 \\ 12 & 3 & 5 \end{vmatrix} \times \begin{vmatrix} 3 & 8 \\ 4 & 9 \\ 4 & 8 \end{vmatrix}$$

$$\begin{vmatrix} 12 & 3 & 4 \\ 9 & 10 & 7 \\ 12 & 3 & 5 \end{vmatrix} \times \begin{vmatrix} 5 \\ 4 \\ 4 \end{vmatrix}$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <MAT> <A*B> <NEW>	MATRIX PRODUCT
Enter the number of rows in matrix A	3 <rA>	rA = 3.
Enter the number of columns in matrix A	3 <cA>	cA = 3.
Enter the number of columns in matrix B	2 <cB>	cB = 2.
Proceed with program	<EOD>	a(1,1)
Enter the A matrix	12 <ENT>	a(2,1)
	9 <ENT>	a(3,1)
	12 <ENT>	a(1,2)
	3 <ENT>	a(2,2)
	10 <ENT>	a(3,2)
	3 <ENT>	a(1,3)

(continued)

## Example: Matrix Products (Continued)

Example (Continued)	Procedure	Press	Display
		4 <ENT>	a(2, 3)
		7 <ENT>	a(3, 3)
		5 <ENT>	EDIT
	Proceed with program	<EOD>	b(1, 1)
	Enter the first column of matrix B	3 <ENT>	b(2, 1)
		4 <ENT>	b(3, 1)
		4 <ENT>	EDIT?
	Proceed with program	<NO>	col = 1.
	View the first column of matrix C	<NXT>	c 1 = 64.
		<NXT>	c 2 = 95.
		<NXT>	c 3 = 68.
	Enter the second column of matrix B	<NXT>	
		8 <ENT>	b(2, 2)
		9 <ENT>	b(3, 2)
		8 <ENT>	EDIT?
	Proceed with program	<NO>	col = 2.
	View the second column of matrix C	<NXT>	c 1 = 155.
		<NXT>	c 2 = 218.

**Example  
(Continued)**

Procedure	Press	Display
	<NXT>	c 3 = 163.
Reuse matrix A	<NXT> <OLD>	
Enter the number of columns in matrix B	1 <CB>	cB = 1.
Proceed with program	<EOD>	b( 1, 1)
Enter the first column of matrix B	5 <ENT>	b( 2, 1)
	4 <ENT>	b( 3, 1)
	4 <ENT>	EDIT?
Proceed with program	<NO>	col = 1.
View matrix C	<NXT>	c 1 = 88.
	<NXT>	c 2 = 113.
	<NXT>	c 3 = 92.

The two resulting C matrices are:

$$\begin{vmatrix} 64 & 155 \\ 95 & 218 \\ 68 & 163 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} 88 \\ 113 \\ 92 \end{vmatrix}$$

# The Inversion/Linear Systems Program

---

This program performs matrix inversion, solves a system of simultaneous equations, or calculates the determinant of a matrix.

---

## Introduction

The Inversion/Linear Systems program has three options that can be used individually or in succession to perform matrix operations. Each option uses the upper-lower decomposition method to perform its operation. The program accepts one  $n^{\text{th}}$  order matrix ( $n$  rows and  $n$  columns).

You must partition the calculator for at least  $11 + n^2 + n$  registers for the inversion or determinant options, or for at least  $11 + n^2 + 3n$  for the simultaneous equations option.

You enter the elements of the matrix and then select an option. The simultaneous equations or determinant options do not affect the stored matrix. However, the inversion option replaces the stored matrix with the result of inversion.

You can save the A matrix as a data file, provided you save the matrix immediately after entering (and, if necessary, editing) it. To save the A matrix, you must specify  $11 + n^2$  data registers, starting with register 000. For information on saving data as a file, refer to the *TI-95 Programming Guide*.

## Reference

*Applied Numerical Analysis*, Curtis F. Gerald, Addison-Wesley Publishing Co., 1970.

## Before Starting the Program

If you have previously saved the A matrix as a file and want to use that data, you must load the data into the data registers before running the Inversion/Linear systems program. For information on loading a data file, refer to the *TI-95 Programming Guide*.

## Starting the Program

To start the Inversion/Linear Systems program:

1. Select **<MAT>** from the **MATHEMATICS** menu.

The program displays the **MATRIX ALGEBRA** menu.

2. Press **<LIN>**.

The program displays:

```
INV/LINEAR SYS
NEW OLD      ESC
```

3. Select the option that applies to your data.

- To enter a new matrix, press **<NEW>** and proceed to "Entering the A Matrix" on the next page.
- To use data you have loaded from the file space or from a storage device, press **<OLD>** and proceed to "Selecting Options" on page 6-15.

### Entering the A Matrix

When you select <NEW> from the **INV/LINEAR SYS** menu, the program displays:

```
n      EOD      ESC
```

To begin entering the A matrix:

1. Enter the number of rows (or simultaneous equations) in the matrix and press <n>.
2. Press <EOD> to proceed.

The program prompts you for the values in the A matrix, one column at a time.

```
a( 1, 1)  
ENT
```

3. Enter all the values, pressing <ENT> after each value.

### The Edit Menu

After you have entered all the values for the A matrix, the program displays:

```
EDIT  
(~) ENT EOD
```

- If you have no changes to make, press <EOD> and proceed to “Selecting Options” on the next page.
- If you want to edit a value in the A matrix, use the editing procedure described at the top of the next page.



# Editing a Value in the A Matrix

To edit a value:

1. Enter the row number of the value and press the  $\boxed{x \sim t}$  key to store the number in the t-register.
2. Enter the column number of the value and press  $\langle i \sim j \rangle$ .

The program displays the current value.

3. Enter the new value and press  $\langle \text{ENT} \rangle$ . (If the displayed value is already correct, press  $\langle \text{ENT} \rangle$ .)
4. Press  $\langle \text{EOD} \rangle$  to leave the **EDIT** menu.

## Selecting Options

After you press  $\langle \text{EOD} \rangle$  to leave the **EDIT** menu, the program displays a menu to let you choose your next option. (If you want to save the A matrix as a data file, you must save it before proceeding with the program.)

```

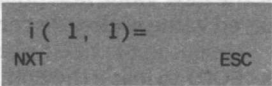
SELECT OPTION
INV  SYS  det      ESC
    
```

- |                              |   |
|------------------------------|---|
| $\langle \text{INV} \rangle$ | Calculates the inverse of the A matrix. For instructions, refer to the next page.                             |
| $\langle \text{SYS} \rangle$ | Lets you enter a constant vector (matrix B) and then solves the system. For instructions, refer to page 6-17. |
| $\langle \text{det} \rangle$ | Calculates the determinant of the A matrix. For instructions, refer to page 6-20.                             |
| $\langle \text{ESC} \rangle$ | Returns to the <b>INV/LINEAR SYS</b> menu.  |

### Calculating the Inverse

To calculate the inverse of the A matrix:

1. Select <INV> from the **SELECT OPTION** menu.
  - ▶ If there is no inverse (the determinant = 0), the program displays the message **SINGULAR**.
  - ▶ If there is an inverse, the program lets you examine each value in the resulting matrix, starting with the elements in the first column.



i ( 1, 1)=  
NXT ESC

The display scrolls to the left until the entire value is in view.



1)=  
NXT ESC

where *xxxx* is the value

2. Press <NXT> to display each remaining value in the matrix.
3. Press <ESC> to return to the **SELECT OPTION** menu.

## Solving the System

To enter a constant vector (matrix B) and solve the system:

1. Select <SYS> from the **SELECT OPTION** menu.

The program prompts you to enter the first value in the constant vector.

```
b( 1)
ENT
```

2. Enter all values in the vector, pressing <ENT> after each value.

After you have entered all the values, the program displays:

```
EDIT?
YES NO
```

3. Decide whether you need to correct any of the values.

- If you have changes to make, proceed to step 4 on the next page.
- If you have no changes to make, proceed to step 6 on page 6-19.

## Solving the System (Continued)

4. To edit the values you have entered for the constant vector, select <YES> from the **EDIT?** menu.

The program displays the same menu you used to enter the values.



```
b( 1)
ENT
```

5. Make any necessary changes in the vector.

- ▶ To display the current value, press the **CE** key.
- ▶ To accept the current value and proceed to the next value, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you have finished editing the column, the program returns to the **EDIT?** menu.

**Solving  
the System  
(Continued)**

6. To display the solutions, select <NO> from the **EDIT?** menu.

- ▶ If there is no solution, the program displays the message **SINGULAR**.
- ▶ If there is a solution, the program displays the solution for each variable, beginning with the first variable.

x 1=  
NXT

xxx  
ESC

where xxx is the value

7. Press <NXT> to display the solution for each remaining variable.
8. When you have displayed all solutions, press <ESC> to return to the **SELECT OPTION** menu.

## Calculating the Determinant

To calculate the determinant of the A matrix:

1. Select <det> from the **SELECT OPTION** menu.

The program displays the value of the determinant.

det=                      xxx        where xxx is the value  
                                      ESC

2. Press <ESC> to return to the **SELECT OPTION** menu.

# Example: Linear Systems

The following example demonstrates how to use the program for matrix inversion and solution of a linear system.

**Example** Solve the linear system shown below.

$$\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}^{-1} \times \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 3 \\ 4 \end{vmatrix}$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <MAT> <LIN> <NEW>	0.
Enter the order of the matrix	2 <n> <EOD>	a(1, 1)
Enter the matrix	2 <ENT>	a(2, 1)
	1 <ENT>	a(1, 2)
	1 <ENT>	a(2, 2)
	3 <ENT>	EDIT?
Proceed with program	<EOD>	SELECT OPTION
Invert matrix	<INV>	1)= 0.6
	<NXT>	1)= -0.2
	<NXT>	2)= -0.2
	<NXT>	2)= 0.4
Proceed with program	<ESC>	SELECT OPTION

(continued)

## Example: Linear Systems (Continued)

The following example demonstrates how to use the program for matrix inversion and solution of a linear system.

Example (Continued)	Procedure	Press	Display
	Solve linear system	<SYS>	b(1)
	Enter constant vector	3 <ENT>	b(2)
		4 <ENT>	EDIT?
	View solutions	<NO>	x 1 = 1.
		<NXT>	x 2 = 1.

(continued)



# The Tridiagonal Systems Program

This program solves a system of simultaneous equations whose coefficients form a tridiagonal matrix. A matrix is tridiagonal when it has nonzero elements in only the central three diagonals.

## Introduction

The Tridiagonal Systems program solves a specialized case of linear systems. To save you the effort of entering many zero elements, the program prompts you for only the elements in the central three diagonals. The program uses the Gaussian elimination method to solve the  $n^{\text{th}}$  order matrix.

You must partition the calculator for at least  $4n + 11$  registers before running the program.

The coefficients of a fourth order tridiagonal system have the following arrangement.

$$\begin{aligned} b_1x_1 + c_1x_2 &= d_1 \\ a_2x_1 + b_2x_2 + c_2x_3 &= d_2 \\ a_3x_2 + b_3x_3 + c_3x_4 &= d_3 \\ a_4x_3 + b_4x_4 &= d_4 \end{aligned}$$

$$\begin{vmatrix} b_1 & c_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & b_4 \end{vmatrix} \times \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{vmatrix}$$

The entry sequence prompts you for the a diagonal, the b diagonal, and the c diagonal. Then you enter the d constant vector. The results are the solution for each variable.

## Reference

*Numerical Methods*, Robert W. Hornbeck, Quantum Publishers, Inc., 1975, pp. 93-98.

## Starting the Program

To start the Tridiagonal Systems program:

1. Select <MAT> from the **MATHEMATICS** menu.

The program displays the **MATRIX ALGEBRA** menu.

2. Press <TRI>.

The program displays:

```
TRIDIAGONAL SYS
n   EOD  ESC
```

3. Enter the order of the system and press <n>.

4. Press <EOD>.

## Entering the "a" Coefficients

After you have entered the order of the system, the program prompts you to enter the "a" coefficients.

```
a 2
ENT
```

Enter the values for all the "a" coefficients, pressing <ENT> after each value.

**The Edit Menu**

After you enter all the "a" coefficients, the calculator displays an **EDIT?** menu that allows you to change any of the values.

```
EDIT?
YES NO
```

- ▶ If you do not want to edit, press <NO> and go to the next page.
- ▶ If you want to edit, press <YES> and use the procedure below.

**Editing the "a" Coefficients**

If you select <YES> from the **EDIT?** menu, the program displays the same menu that you used to enter the values.

```
a 2
ENT
```

- ▶ To display the current value, press the **[CE]** key.
- ▶ To accept the current value and proceed to the next value, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you press <ENT> to accept or edit the last value, the program returns to the **EDIT?** menu.

## Entering the Remaining Coefficients

After you enter the "a" coefficients and select <NO> from the **EDIT?** menu, the program prompts you to enter the "b" coefficients.

```
b 1
ENT
```

1. Use the same procedure you used to enter the "a" coefficients.

After you have entered and, if necessary, edited the "b" coefficients, the program prompts you to enter the "c" coefficients.

```
c 1
ENT
```

2. Use the same procedure to enter the "c" coefficients.

After you have entered and, if necessary, edited the "c" coefficients, the program prompts you to enter the "d" coefficients.

```
d 1
ENT
```

3. Use the same procedure to enter the "d" coefficients.

The following example demonstrates how to use the Tridiagonal Systems program.

## Displaying Results

After you enter all the data and select <NO> from the **EDIT?** menu, the program proceeds in one of two ways.

- ▶ If the program cannot solve the system, it displays the message **"SINGULAR."** (The quotes indicate that the program is not able to solve the system, although the system may have a solution.)
- ▶ If the program is able to solve the system, it displays the solutions for the variables, beginning with the first variable.

x 1=  
NXT

xxx  
ESC

where *xxx* is the value

1. Press <NXT> to display the solution for each remaining variable.
2. When you have examined all the solutions, press <ESC> to return to the **TRIDIAGONAL SYS** menu.

## Example: Tridiagonal Systems

The following example demonstrates how to use the Tridiagonal Systems program.

**Example** Find the solutions to the following tridiagonal matrix.

$$\begin{vmatrix} 6 & 4 & 0 \\ 8 & 14 & 6 \\ 0 & 2 & 18 \end{vmatrix} \times \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 27 \\ 86 \\ 78 \end{vmatrix}$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <MAT> <TRI>	TRIDIAGONAL SYS
Enter order of the matrix	3 <n> <EOD>	a 2
Enter the a diagonal	8 <ENT> 2 <ENT>	a 3 EDIT?
Proceed with program	<NO>	b 1
Enter the b diagonal	6 <ENT> 14 <ENT> 18 <ENT>	b 2 b 3 EDIT?
Proceed with program	<NO>	c 1
Enter the c diagonal	4 <ENT> 6 <ENT>	c 2 EDIT?
Proceed with program	<NO>	d 1

Example  
(Continued)

Procedure	Press	Display
Enter the d vector	27 <ENT>	d 2
	86 <ENT>	d 3
	78 <ENT>	EDIT?
Display each result	<NO>	x 1 = 2.5
	<NXT>	x 2 = 3.
	<NXT>	x 3 = 4.

The solutions to the matrix are 2.5, 3, and 4.

# The Eigenvalues Program

This program determines the eigenvalues and eigenvectors for a symmetric matrix.

## Introduction

The Eigenvalues program uses the Jacobi method to generate the eigenvalues. An  $n^{\text{th}}$  order matrix has  $n$  eigenvalues. Each eigenvalue is associated with an  $n$ -element eigenvector.

Before running the program, you must partition the calculator for at least  $26 + n^2 + (n + 1)^2$  registers.

The symmetry requirement allows the program to prompt you only for elements above and along the main diagonal. You enter all of the first row. Enter the second through last elements of the second row. Enter the third through last elements of the third row. Continue this pattern until you enter the last element of the last row. The entry sequence for third and fourth order matrices is illustrated below.

1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
—	4 <sup>th</sup>	5 <sup>th</sup>	—	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>
—	—	6 <sup>th</sup>	—	—	8 <sup>th</sup>	9 <sup>th</sup>
			—	—	—	10 <sup>th</sup>

The lower left portion must be symmetric to the upper right portion.

## Reference

*Numerical Methods*, Robert W. Hornbeck, Quantum Publishers, Inc., 1975.



**Starting  
the Program**

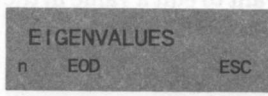
To start the Eigenvalues program:

1. Select <MAT> from the **MATHEMATICS** menu.

The calculator displays the **MATRIX ALGEBRA** menu.

2. Press <EIG>.

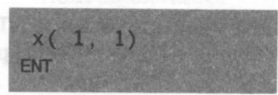
The program displays:



3. Enter the size of the matrix and press <n>.

- 4 Press <EOD>.

When you press <EOD>, the program displays a menu that lets you enter the values in the upper right portion of the matrix.



Enter all the values, pressing <ENT> after each value.

When you have entered all the values, you can edit any value you may have entered incorrectly.

**Entering  
the Matrix**

## The Eigenvalues Program (Continued)

---

### The Edit Menu

After you enter all the values, the program displays:

```
EDIT
i~j ENT EOD
```

- ▶ If you have no changes to make, press <EOD> and proceed to the next page.
- ▶ If you want to edit a value in the matrix, use the procedure described below.

### Editing a Value in the Matrix

To edit a value:

1. Enter the row number of the value and press the  $\boxed{x\sim t}$  key to store the number in the t-register.
2. Enter the column number of the value and press <i~j>.

The program displays the current value.

3. Enter the new value and press <ENT>. (If the displayed value is already correct, press <ENT>.)
4. Press <EOD> to leave the **EDIT** menu.

## Displaying the Results

When you select <EOD> from the **EDIT** menu, the program displays the first eigenvalue.

```
L 1=      xxx
NXT      ESC
```

where *xxx* is the value

1. Press <NXT> to display the first component of the eigenvector.

The program displays:

```
1=      xxx
NXT      ESC
```

where *xxx* is the component

2. Press <NXT> repeatedly to display each remaining component.

When all components for the current eigenvector have been displayed, pressing <NXT> displays the next eigenvalue.

3. Repeat steps 1 and 2 until all eigenvalues and their components have been displayed.
4. Press <ESC> to return to the **EIGENVALUES** menu.

## Example: Eigenvalues

The following example demonstrates how to use the Eigenvalues program.

**Example** Find the eigenvalues and eigenvectors of the following matrix.

0.874995	0.4558279	-0.6873331
0.4558279	1.210105	-2.03125
-0.6873331	-2.03125	10.78151

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <MAT> <EIG>	EIGENVALUES
Enter order of the matrix	3 <n> <EOD>	x(1, 1)
Enter the elements	.874995 <ENT>	x(1, 2)
	.4558279 <ENT>	x(1, 3)
	.6873331 <b>+/-</b> <ENT>	x(2, 2)
	1.210105 <ENT>	x(2, 3)
	2.03125 <b>+/-</b> <ENT>	x(3, 3)
	10.78151 <ENT>	EDIT
Proceed with results	<EOD>	L 1 = 1.116763275
View components of the first eigenvector	<NXT>	1 = 0.717443239
	<NXT>	2 = .6696997915
	<NXT>	3 = .1917743155

This chapter tells you how to use the Polynomial Product program to multiply two or more polynomials.

## Example (Continued)

Procedure	Press	Display
View second eigenvalue	<NXT>	L 2 = 0.498740552
View components of the second eigenvector	<NXT>	1 = -.6927248751
	<NXT>	2 = .7149279642
	<NXT>	3 = .0949223547
View third eigenvalue	<NXT>	L 3 = 11.2511062
View components of the third eigenvector	<NXT>	1 = -.0735353398
	<NXT>	2 = -.2009482403
	<NXT>	3 = .9768379387



# Chapter 7: Multiplying Polynomials

This chapter tells you how to use the Polynomial Product program to multiply two or more polynomials.

Table of Contents	The Polynomial Product Program	7-2
	Example: Polynomial Products	7-7

# The Polynomial Product Program

---

This program multiplies two polynomials.

---

## Introduction

The Polynomial Product program lets you enter two polynomials, called  $P_A$  and  $P_B$ . The program then multiplies  $P_A$  by  $P_B$  to result in  $P_C$ . The order of  $P_A$  plus the order of  $P_B$  equals the order of  $P_C$ . Each polynomial is in the form

$$P = a_n x^n + \dots + a_3 x^3 + a_2 x^2 + a_1 x + a_0.$$

You must partition the calculator for 125 registers before running the program.  $P_C$  cannot exceed 95<sup>th</sup> order,  $P_B$  cannot exceed 20<sup>th</sup> order, and  $P_A$  cannot exceed (95 - order of  $P_B$ ) order.

After the program multiplies the polynomials, you can view the coefficients of  $P_C$ .  $P_C$  becomes the new  $P_A$ . If you elect to chain multiplications, the program prompts you for another  $P_B$ , which is multiplied by the previous result. If you elect not to chain, the program prompts you for both a new  $P_A$  and a new  $P_B$ .

## Reference

*Advanced Engineering Mathematics*, C. R. Wylie, Jr., McGraw-Hill Book Company, 1966.



## Starting the Program

To start the Polynomial Product program:

1. Select  $\langle *P \rangle$  from the **MATHEMATICS** menu.

The program displays:

```
POLYNOM PRODUCT
dgA EOD
```

2. Enter the degree of the first polynomial (polynomial A) and press  $\langle dgA \rangle$ .
3. Press  $\langle EOD \rangle$  to proceed.

The program prompts you to enter the coefficient of each term in the polynomial, beginning with the highest-order term.

```
A x
ENT
```

where  $x$  is the order of the term

4. Enter the coefficient and press  $\langle ENT \rangle$ .

The program then prompts you to enter the coefficient for the next term.

5. Repeat step 4 until all of the coefficients in polynomial A are entered.

The program then displays the menu shown at the top of the next page.

## The Edit Menu

After you enter all the coefficients, the program displays an **EDIT?** menu that allows you to change any of the values.

```
EDIT?  
YES NO
```

- ▶ If you do not want to edit, press <NO> and go to the next page.
- ▶ If you want to edit, press <YES> and follow the procedure below.

## Editing the Coefficients

When you select <YES> from the **EDIT?** menu, the program displays the same menu that you used to enter the coefficients.

```
A x  
ENT
```

where  $x$  is the order of the term

- ▶ To display the current value of the coefficient, press the **CE** key.
- ▶ To accept the current value and proceed to the next coefficient, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you press <ENT> to accept or edit the value of the last coefficient,  $a_0$ , the program returns to the **EDIT?** menu.

## Entering the B Polynomial

After you enter the A polynomial and select <NO> from the **EDIT?** menu, the program prompts you to begin entering the B polynomial.

```
POLYNOM PRODUCT
dgB EOD
```

Using the procedures shown on pages 7-3 and 7-4, enter (and edit, if necessary) the degree of the B polynomial and each of its coefficients.

## Viewing the Results

After you enter the B polynomial and select <NO> from the **EDIT?** menu, the program multiplies the two polynomials and displays the coefficient of the highest order term in the result.

```
C x
NXT          ESC
```

where  $x$  is the order of the resulting polynomial

- To display the next coefficient, press <NXT>. (When **C0** is displayed, pressing <NXT> repeats the coefficients.)
- To proceed to the **CHAIN?** menu (described on the next page), press <ESC>.

### The Chain Menu

When you press <ESC> while a coefficient is displayed, the program displays a menu that enables you to perform chain calculations.



```
CHAIN?  
YES NO
```

- To restart the program and begin a new calculation, press <NO>.
- To perform a chain calculation, press <YES>.

### Performing a Chain Calculation

When you select <YES> from the **CHAIN?** menu, the program prompts you to enter a new B polynomial, as described on the previous page. (The result of the previous calculation is automatically used as the new A polynomial.)

After entering the B polynomial, you can:

- Display the coefficients of the chained result.
- Perform an additional chain calculation.

## Example: Polynomial Products

The example below demonstrates how to use the Polynomial Products program.

**Example** Multiply the following polynomials.

$$(3x^3 - 2x^2)(13x^2 - 2x - 3)(9x^5 - 3x^3)$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <*P>	POLYNOM PRODUCT
Enter the degree of $P_A$	3 <dgA> <EOD>	A 3
Enter the A coefficients	3 <ENT>	A 2
	2 <b>+/-</b> <ENT>	A 1
	0 <ENT>	A 0
	0 <ENT>	EDIT?
Proceed with program	<NO>	
Enter the degree of $P_B$	2 <dgB> <EOD>	B 2
Enter the B coefficients	13 <ENT>	B 1
	2 <b>+/-</b> <ENT>	B 0
	3 <b>+/-</b> <ENT>	EDIT?
View each C coefficient	<NO>	C 5 = 39.
	<NXT>	C 4 = -32.
	<NXT>	C 3 = -5.
	<NXT>	C 2 = 6.
	<NXT>	C 1 = 0.

(continued)

# Example: Polynomial Products (Continued)

Example (Continued)	Procedure	Press		Display
		<b>&lt;NXT&gt;</b>	<b>C 0 =</b>	
	Proceed with program	<b>&lt;ESC&gt;</b>	<b>CHAIN?</b>	
	Chain with another multiplication	<b>&lt;YES&gt;</b>	<b>POLYNOM PRODUCT</b>	
	Enter the degree of $P_B$	<b>5 &lt;dgB&gt;</b>		
		<b>&lt;EOD&gt;</b>	<b>B 5</b>	
	Enter the B coefficients	<b>9 &lt;ENT&gt;</b>	<b>B 4</b>	
		<b>0 &lt;ENT&gt;</b>	<b>B 3</b>	
		<b>3 <span style="border: 1px solid black; padding: 0 2px;">+/-</span> &lt;ENT&gt;</b>	<b>B 2</b>	
		<b>0 &lt;ENT&gt;</b>	<b>B 1</b>	
		<b>0 &lt;ENT&gt;</b>	<b>B 0</b>	
		<b>0 &lt;ENT&gt;</b>	<b>EDIT?</b>	
	View each C coefficient	<b>&lt;NO&gt;</b>	<b>C10 =</b>	<b>351.</b>
		<b>&lt;NXT&gt;</b>	<b>C 9 =</b>	<b>- 288.</b>
		<b>&lt;NXT&gt;</b>	<b>C 8 =</b>	<b>- 162.</b>
		<b>&lt;NXT&gt;</b>	<b>C 7 =</b>	<b>150.</b>
		<b>&lt;NXT&gt;</b>	<b>C 6 =</b>	<b>15.</b>
		<b>&lt;NXT&gt;</b>	<b>C 5 =</b>	<b>- 18.</b>
		<b>&lt;NXT&gt;</b>	<b>C 4 =</b>	<b>0.</b>

The remaining coefficients are zero.

# Chapter 8: Finding Roots of a Function

---

This chapter describes how to use the programs available through the <ZRO> selection of the MATHEMATICS menu.

---

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Four programs are available from the FUNCTION ZEROS menu.

---

## The Function Zeros Menu

When you select <ZRO> from the menu, the calculator displays:

```
FUNCTION ZEROS
Q-D BAI BIS NTN
```

- <Q-D>      Uses the Q-D method to approximate all the roots of a polynomial. This program is generally used to provide initial guesses for the Bairstow root-finding method.
- <BAI>      Uses the Bairstow method to find the exact roots of a polynomial.
- <BIS>      Uses the bisection method to find a root between two boundaries for a function you have stored in program memory.
- <NTN>      Uses Newton's method to find a root of a function you have stored in program memory.



This program approximates the roots of a polynomial. The approximate roots are helpful as guesses for the Bairstow method.

---

## Introduction

The Q-D method program approximates the roots of a polynomial whose coefficients you enter. It is able to identify the roots without an initial estimate but requires many iterations to achieve accuracy comparable to the Bairstow method. Because the Bairstow method can fail to converge for unrealistic initial guesses, the Q-D method is recommended for identifying initial guesses for use in the Bairstow program.

The program can usually determine the roots with sufficient accuracy for the Bairstow program in 20 to 30 iterations. Finding roots with four-digit accuracy can take more than 100 iterations.

Before running the program, you must partition the calculator for at least 125 data registers, which is the default partitioning.

Because the calculator has built-in solutions for quadratic and cubic polynomials, the program requires you to enter a polynomial of fourth order or higher. The largest polynomial the program can use is 25th order.

The polynomial you enter has the form

$$0 = a_n x^n + \dots + a_3 x^3 + a_2 x^2 + a_1 x + a_0.$$

The program generates values labeled e and q. To correctly interpret these values, you should be familiar with the principles of the Q-D method of finding roots.

## Reference

*Applied Numerical Analysis*, Curtis F. Gerald, Addison-Wesley, 1970.

### Starting the Program

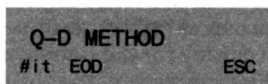
To start the program that uses the Q-D method:

1. Select <ZRO> from the **MATHEMATICS** menu.

The calculator displays the **FUNCTION ZEROS** menu.

2. Press <Q-D>.

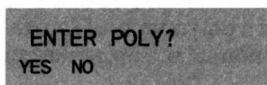
The program displays:



```
Q-D METHOD
#it EOD      ESC
```

3. Enter the number of iterations you want performed and press <#it>.
4. Press <EOD>.

The program displays:



```
ENTER POLY?
YES NO
```

5. Select the appropriate option.
  - ▶ To enter a new polynomial, press <YES> and proceed to "Entering the Coefficients" on the next page.
  - ▶ To use a polynomial you have already entered (using either the Bairstow program or a previous run of this program), press <NO> and proceed to "Displaying the Approximate Roots" on page 8-7.

---

## Entering the Coefficients

When you select <YES> from the **ENTER POLY?** menu, the program displays:

```
ENTER DEGREE
n      EOD      ESC
```

1. Enter the degree of the polynomial and press <n>.
2. Press <EOD>.

The program prompts you to enter the coefficient of each term, beginning with the first term.

```
A x
ENT
```

where  $x$  is the number of the term

3. Enter the coefficient and press <ENT>.
4. Repeat step 3 until all the coefficients are entered.

The program then displays the menu shown on the next page.

### The Edit Menu

After you enter all the coefficients, the program displays an **EDIT?** menu that allows you to change any of the values.



```
EDIT?  
YES NO
```

- ▶ If you do not want to edit, press <NO> and go to the next page. The computation time for the approximate roots in some cases can be several minutes.
- ▶ If you want to edit, press <YES> and use the procedure below.

### Editing the Coefficients

When you select <YES> from the **EDIT?** menu, the program displays the same menu that you used to enter the coefficients.



```
A x  
ENT
```

where  $x$  is the number of the term

- ▶ To display the current value of the coefficient, press the **CE** key.
- ▶ To accept the current value and proceed to the next coefficient, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you press <ENT> to accept or edit the value of the last coefficient, the program returns to the **EDIT?** menu.

---

## Displaying the Approximate Roots

After you select <NO> from either the **ENTER POLY?** or the **EDIT?** menu, the program displays the value of the first  $e$ . (If you are not using a printer and you plan to compute values for  $r$  and  $s$ , keep a written record of the  $e$ 's and  $q$ 's.)

```
e 1=      xxx      where xxx is the value
NXT
```

1. Press <NXT>.

The program displays the corresponding value of  $q$ . Each  $q$  is an approximate root.

```
q 1=      xxx      where xxx is the value
NXT
```

2. Press <NXT>.

- ▶ If there are remaining values for  $e$  and  $q$ , the program displays the next value of  $e$ .
- ▶ If all values of  $e$  and  $q$  have been displayed, the program displays the menu shown on the next page.

To correctly interpret the  $e$ 's and  $q$ 's, you must examine them after an initial number of iteration, and again after more iterations. From the trends evident in checking these values twice (or more), you can infer whether the roots are real or complex.

If all the  $e$ 's are approaching zero, it is likely that all roots are real, and the  $q$ 's are the approximate roots. If an  $e$  is not approaching zero, it implies that a pair of complex roots is present.

### Selecting Options

After the program has displayed all the values of  $e$  and  $q$ , it displays:

```
SELECT OPTIONS
Q-D  r/s          ESC
```

- <Q-D>      Lets you perform additional iterations.
- <r/s>      Lets you compute the  $r$  and  $s$  values for use in the Bairstow program.
- <ESC>      Returns to the **Q-D METHOD** menu.

### Performing Additional Iterations

To perform additional iterations using the Q-D method:

1. Select the <Q-D> option.

The program displays:

```
CONTINUE Q-D
#it EOD          ESC
```

2. Enter the number of additional iterations you want performed and press <#it>.
3. Press <EOD>.

The program displays the approximate roots, as described on the previous page.

---

## Computing the r and s values

If you are using the Q-D program to compute r and s values for use in the Bairstow program, follow these steps.

1. Choose a pair of roots from the list of e and q values.
2. Select the <r/s> option.

The program displays:

```
COMPUTE r, s
1st 2nd EOD      ESC
```

3. Enter the number of one of the roots you have chosen and press <1st>. For example, if the root is the third in the list, enter 3.
4. Enter the number of the other root and press <2nd>.
5. Press <EOD> to display the r value.

```
r=      xxx
NXT
```

where xxx is the value

6. Press <NXT> to see the s value.
7. Press <NXT> again to return to the **COMPUTE r, s** menu.

## Examples: The Q-D Method

The examples below demonstrate how to use the Q-D method program.

### Real Roots Example

Find the approximate roots of the polynomial

$$0 = 4x^4 - 32x^3 + 85x^2 - 93x + 36.$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <ZRO> <Q-D>	Q-D METHOD
Enter the number of iterations	40 <#it> <EOD>	ENTER POLY?
Select new polynomial	<YES>	ENTER DEGREE
Enter the degree of the polynomial	4 <n> <EOD>	A 4
Enter the coefficients	4 <ENT>	A 3
	32 <b>+/-</b> <ENT>	A 2
	85 <ENT>	A 1
	93 <b>+/-</b> <ENT>	A 0
	36 <ENT>	EDIT?
Proceed with program	<NO>	e 1 = - 7.717284 - 16
	<NXT>	q 1 = 4.
	<NXT>	e 2 = - .0010444508
	<NXT>	q 2 = 1.539062498



**Real Roots  
Example  
(Continued)**

Procedure	Press	Display
	<NXT> e 3 =	-.0000002255
	<NXT> q 3 =	1.460937989
	<NXT> e 4 =	0.
	<NXT> q 4 =	.9999995128
	<NXT> SELECT OPTION	
Compute r and s values	<r/s> COMPUTE r, s	
Specify first and second roots to generate r and s	1 <1st> 1st =	1.
	2 <2nd> 2nd =	2.
View r and s	<EOD> r =	-5.539062498
	<NXT> s =	6.156249992

The approximate roots are q1, q2, q3, and q4. The actual roots are 4, 1.5, 1.5, and 1, which could be determined by using more iterations or using this program's values for r and s with the Bairstow method program.

## Complex Roots Example

Find the approximate roots of the polynomial

$$0 = 4x^4 - 20x^3 + 25x^2 - 45x + 36.$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <ZRO> <Q-D>	Q-D METHOD
Enter the number of iterations	40 <#it> <EOD>	ENTER POLY?
Select new polynomial	<YES>	ENTER DEGREE
Enter the degree of the polynomial	4 <n> <EOD>	A 4
Enter the coefficients	4 <ENT>	A 3
	20 <b>+/-</b> <ENT>	A 2
	25 <ENT>	A 1
	45 <b>+/-</b> <ENT>	A 0
	36 <ENT>	EDIT?
Proceed with program	<NO>	e 1 = - 1.625082 - 17
	<NXT>	q 1 = 4.
	<NXT>	e 2 = 3.16333336
	<NXT>	q 2 = 2.08333329

**Complex Roots  
Example  
(Continued)**

Procedure	Press	Display
	<NXT>	e 3 = .0000001342
	<NXT>	q 3 = -2.083333204
	<NXT>	e 4 = 0.
	<NXT>	q 4 = .9999999141
	<NXT>	SELECT OPTION
Compute r and s values	<r/s>	COMPUTE r, s
Specify second and third roots to generate r and s	2 <1st>	1st = 2.
	3 <2nd>	2nd = 3.
View r and s	<EOD>	r = -.0000000859
	<NXT>	s = 2.250000048

The approximate roots are q1, q2, q3, and q4. The actual roots are 4, 1.5i, -1.5i, and 1, which could be determined using this program's values for r and s with the Bairstow method program.

# The Bairstow Method

---

This program approximates the roots of a polynomial. It finds the precise roots quickly, provided that initial guesses (available from the Q-D program) are reasonably close to the actual roots.

---

## Introduction

The Bairstow method program finds the roots of a polynomial whose coefficients you enter. It is able to converge on the roots more quickly than the Q-D method but requires an initial guess for each root. Because the Q-D method can generate the initial guesses, the Q-D method is helpful when the roots you guess fail to make the Bairstow program converge on the precise roots.

Before running the program, you must partition the calculator for at least 125 data registers, which is the default partitioning.

The polynomial you enter has the form

$$0 = a_n x^n + \dots + a_3 x^3 + a_2 x^2 + a_1 x + a_0.$$

Because the calculator has built-in solutions for quadratic and cubic polynomials, the program requires you to enter a polynomial of fourth order or higher. The largest polynomial the program can use is 25<sup>th</sup> order.

The program reduces the polynomial by a quadratic factor,  $x^2 + rx + s = 0$ , resulting in a polynomial whose degree is two less than the original polynomial. You enter  $r$  and  $s$ , and the program searches for the remaining roots. You can determine the values of  $r$  and  $s$  from the Q-D program or you can estimate the values of  $r$  and  $s$ .

## Reference

*Applied Numerical Analysis*, Curtis F. Gerald, Addison-Wesley, 1970.

To start the program that uses the Bairstow method:

1. Select <ZRO> from the **MATHEMATICS** menu.

The calculator displays the **FUNCTION ZEROS** menu.

2. Press <BAI>.

The program displays:

```
BAIRSTOW METHOD
#it err EOD      ESC
```

3. Enter the number of iterations you want performed and press <#it>.
4. Enter the allowable error and press <err>.
5. Press <EOD>.

The program displays:

```
ENTER POLY?
YES NO
```

6. Select the appropriate option.
  - ▶ To enter a new polynomial, press <YES> and proceed to "Entering the Coefficients" on the next page.
  - ▶ To use a polynomial you have already entered (using either the Q-D program or a previous run of this program), press <NO> and proceed to "Entering the r and s Values" on page 8-18.

### Entering the Coefficients

When you select <YES> from the **ENTER POLY?** menu, the program displays:

```
ENTER DEGREE
n      EOD      ESC
```

1. Enter the degree of the polynomial and press <n>.
2. Press <EOD>.

The program prompts you to enter the coefficient of each term, beginning with the first term.

```
A x
ENT
```

where  $x$  is the number of the term

3. Enter the coefficient and press <ENT>.
4. Repeat step 3 until all the coefficients are entered.

The program then displays the menu shown on the next page.

---

## The Edit Menu

After you enter all the coefficients, the program displays an **EDIT?** menu that allows you to change any of the values.



```
EDIT?  
YES NO
```

- ▶ If you do not want to edit, press <NO> and go to the next page.
- ▶ If you want to edit, press <YES> and use the procedure below.

## Editing the Coefficients

When you select <YES> from the **EDIT?** menu, the program displays the same menu that you used to enter the coefficients.



```
A x  
ENT
```

where  $x$  is the number of the term

- ▶ To display the current value of the coefficient, press the **CE** key.
- ▶ To accept the current value and proceed to the next coefficient, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you press <ENT> to accept or edit the value of the last coefficient, the program returns to the **EDIT?** menu.

### Entering the r and s Values

When you select <NO> from either the **ENTER POLY?** or the **EDIT?** menu, the program displays:

```
ENTER x^2+rx+s=0
r      s      EOD      ESC
```

1. Enter the r value and press <r>.
2. Enter the s value and press <s>.
3. Press <EOD>.

The program proceeds in one of two ways.

- ▶ If the reduced polynomial is not cubic or quadratic, the program finds the roots of the factor  $x^2 + rx + s$ , allows you to examine the reduced polynomial, and proceeds with another reduction.
  - ▶ If the reduced polynomial is cubic or quadratic, the program finds the roots.
4. Proceed according to the message that is displayed.
    - ▶ If the message is **REAL ROOTS**, go to the next page.
    - ▶ If the message is either **COMPLEX ROOTS** or **1 REAL, 2 COMPLEX**, go to the applicable section on page 8-20.

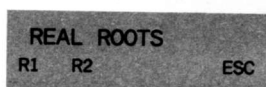


---

## Real Roots

Use the display on this page that matches the display shown on your calculator.

If the polynomial is not cubic, the program displays:



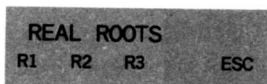
A rectangular display area with a dark background and light text. The text is arranged in two lines. The first line contains the words "REAL ROOTS" in all caps. The second line contains "R1", "R2", and "ESC" in all caps, with "R1" and "R2" separated by a space and "ESC" separated by a larger space.

1. Press  $\langle R1 \rangle$  and  $\langle R2 \rangle$  to display the two real roots.
2. Press  $\langle ESC \rangle$ .

The program proceeds in one of two ways.

- ▶ If the roots are not the last two roots of the polynomial, the program displays **SEE NEXT POLY?**. Use the procedures described on page 8-21.
- ▶ If all roots have been found, the program returns to the **FUNCTION ZEROS** menu.

If the reduced polynomial is cubic, the program displays:



A rectangular display area with a dark background and light text. The text is arranged in two lines. The first line contains the words "REAL ROOTS" in all caps. The second line contains "R1", "R2", "R3", and "ESC" in all caps, with "R1", "R2", and "R3" separated by spaces and "ESC" separated by a larger space.

1. Press  $\langle R1 \rangle$ ,  $\langle R2 \rangle$ , and  $\langle R3 \rangle$  to display the three real roots.
2. Press  $\langle ESC \rangle$  to return to the **FUNCTION ZEROS** menu.

### Complex Roots

If the roots of the equation are a complex conjugate pair, the program displays:

```
COMPLEX ROOTS
Re  Im          ESC
```

1. Press <Re> and <Im> to display the real and the imaginary parts of the complex conjugate pair.
2. Press <ESC>.

The program proceeds in one of two ways.

- ▶ If the roots are not the last two roots of the polynomial, the program displays **SEE NEXT POLY?**. Use the procedures described on the next page.
- ▶ If all roots have been found, the program returns to the **FUNCTION ZEROS** menu.

### 1 Real, 2 Complex

If the equation has one real root and a complex conjugate pair of roots, the program displays:

```
1 REAL, 2 COMPLEX
Re  Im  R3    ESC
```

1. Press <R3> to display the real root.
2. Press <Re> and <Im> to display the real and the imaginary parts of the complex conjugate pair.
3. Press <ESC> to return to the **FUNCTION ZEROS** menu.

## Selecting Options

If the roots displayed are not the last roots of the polynomial, the program displays a menu to let you view the coefficients of the reduced polynomial.

```
SEE NEXT POLY?
YES NO      ESC
```

- ▶ To return to the **ENTER POLY?** menu without viewing the coefficients, press **<NO>**.
- ▶ To view the coefficients, press **<YES>** and follow the procedure below.

## Displaying the Reduced Polynomial

When you press **<YES>**, the program displays the coefficients of the reduced polynomial, beginning with the coefficient of the highest-order term.

```
A x=      xxx
NXT
```

where  $x$  is the order and  $xxx$  is the value

Press **<NXT>** repeatedly to display each remaining coefficient.

After all the coefficients have been displayed, pressing **<NXT>** returns you to the **SEE NEXT POLY?** menu.

## Example 1: The Bairstow Method

The following example demonstrates how to use the Bairstow method program.

### Example

Find the roots of the polynomial

$$x^5 - 17.8x^4 + 99.41x^3 - 261.218x^2 + 352.611x - 134.106 = 0.$$

Limit the program to 50 iterations, specify an allowable error of .00001, and use zero for the initial guesses.

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <ZRO> <BAI>	BAIRSTOW METHOD
Enter the number of iterations allowed	50 <#it>	#it = 50
Enter the allowable error	.00001 <err>	err = 0.00001
Proceed with program	<EOD>	ENTER POLY?
Select new polynomial	<YES>	ENTER DEGREE
Enter the degree of the polynomial	5 <n> <EOD>	A 5
Enter the coefficients	1 <ENT>	A 4
	17.8 <b>+/-</b> <ENT>	A 3
	99.41 <ENT>	A 2
	261.218 <b>+/-</b> <ENT>	A 1
	352.611 <ENT>	A 0

**Example  
(Continued)**

Procedure	Press	Display
	134.106 $\boxed{+/-}$ <ENT>	EDIT?
Proceed with program	<NO>	ENTER $x^2 + rx + s = 0$
Enter initial r and s	0 <r> 0 <s> <EOD>	REAL ROOTS
View the roots	<R1>	R1 = 3.619868415
	<R2>	R2 = .5801315846
Proceed with program	<ESC>	SEE NEXT POLY?
View the coefficients of the reduced polynomial	<YES>	A 3 = 1.
	<NXT>	A 2 = -13.6
	<NXT>	A 1 = 40.19
	<NXT>	A 0 = -63.86000001

## Example 2: Q-D and Bairstow Combination

The following example demonstrates how to use the Q-D method in conjunction with the Bairstow method.

### Example

Use a combination of the Q-D and Bairstow methods to find the roots of the polynomial

$$0 = x^5 + 13x^4 + 62x^3 + 118x^2 + 97x + 29.$$

Procedure	Press	Display
Select the Q-D program	<b>RUN</b> <MTH> <--> <ZRO> <Q-D>	Q-D METHOD
Enter the number of iterations	25 <#it> <EOD>	ENTER POLY?
Select new polynomial	<YES>	ENTER DEGREE
Enter the degree of the polynomial	5 <n> <EOD>	A 5
Enter the coefficients	1 <ENT>	A 4
	13 <ENT>	A 3
	62 <ENT>	A 2
	118 <ENT>	A 1
	97 <ENT>	A 0
	29 <ENT>	EDIT?
Proceed with program	<NO>	e 1 = - 21.2602515

Example  
(Continued)

Procedure	Press	Display
	<NXT> q 1 =	- 13.98747053
	<NXT> e 2 =	1.645003 - 16
	<NXT> q 2 =	3.987470526
	<NXT> e 3 =	.0031824311
	<NXT> q 3 =	- 1.07817781
	<NXT> e 4 =	.0029307628
	<NXT> q 4 =	- 0.996959403
	<NXT> e 5 =	0.
	<NXT> q 5 =	- .9248627866
	<NXT> SELECT OPTION	
Compute r and s values	<r/s> COMPUTE r, s	
Specify first and second roots to generate r and s	1 <1st> 1st =	1.
	2 <2nd> 2nd =	2.
View r and s	<EOD> r =	10.
	<NXT> s =	29.

(continued)

## Example (Continued)

Procedure	Press	Display
Select the Bairstow program	<NXT> <ESC> <ESC> <ESC> <BAI>	BAIRSTOW METHOD
Enter the number of iterations allowed	25 <#it>	#it = 25
Enter the allowable error	.00000001 <err>	err = 0.00000001
Proceed with program	<EOD>	ENTER POLY?
Proceed with program	<NO>	ENTER $x^2 + rx + s = 0$
Enter initial r and s	10 <r> 29 <s> <EOD>	COMPLEX ROOTS



**Example  
(Continued)**

Procedure	Press	Display
View the real part	<Re>	Re = -5.
View the imaginary part	<Im>	Im = 2.
Proceed with program	<ESC>	SEE NEXT POLY?
	<NO>	ENTER POLY?
	<NO>	ENTER $x^2 + rx + s = 0$
Use current values of r and s	<EOD>	REAL ROOTS
View the roots	<R1>	R1 = -1.
	<R2>	R2 = -1.
	<R3>	R3 = -1.

The roots of the reduction polynomial are  $-5 + 2i$  and  $-5 - 2i$ . The roots of the remaining polynomial are all  $-1$ .

# The Bisection Method

---

This program searches for the root of a function that crosses the axis between two bounds.

---

## Introduction

The bisection method requires two bounds at which a function has opposite signs. It checks the sign at the midpoint of the interval to decide where to search for the root next. Each midpoint check bisects the interval. This process continues until the size of the interval is less than the allowable error.

You must store the function as a subroutine labeled *fx* before running the program. When the program calls the subroutine, the value for the independent variable is available in both the display and data register A (000).

The function must have a value at the lower bound that is opposite in sign from the value of the function at the upper bound. This requirement prevents the program from finding a root at which the function is tangent to the axis.

---

For the  
Beginning  
Programmer

Before using the bisection method program, you must store the function as a subroutine in program memory.

If you are not familiar with keystroke programming, refer to the following chapters in the *TI-95 Programming Guide* for instructions.

- ▶ “Working with Programs on the TI-95”
- ▶ “Using Calculator Keystrokes in a Program”
- ▶ “Controlling the Sequence of Operations”

Rules for  
Storing  
the Function

When you store the function as a subroutine, follow these rules.

- ▶ The subroutine must be labeled  $fx$  (lowercase only).
- ▶ Anytime the subroutine needs the independent variable, use a RCL A or RCL 000 instruction.
- ▶ The subroutine must terminate with a RTN instruction.

### Starting the Program

To start the program that uses the bisection method:

1. Store the function as a subroutine, following the rules listed on the previous page.
2. If the subroutine involves any of the trig functions, select the angle units you want to use.
3. Select <ZRO> from the **MATHEMATICS** menu.

The calculator displays the **FUNCTION ZEROS** menu.

4. Press <BIS>.

The program displays:

```
BISECTION METHOD
LO HI  err EOD ESC
```

5. Enter the lower limit of  $x$  and press <LO>.
6. Enter the upper limit of  $x$  and press <HI>.
7. Enter the allowable error and press <err>.

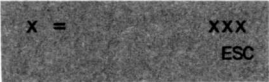
---

## Finding the Root

After you have entered the parameters for the program, you can find the root of the function stored in program memory.

1. Press  $\langle \text{EOD} \rangle$ . The program searches for the root.

If the program finds a root, it is displayed.

 where  $xxx$  is the root  
ESC

If the values of the function at the limits indicate no root is in the interval, the message **INVALID LIMITS** is displayed.

2. Press  $\langle \text{ESC} \rangle$ .

The program returns to the **BISECTION METHOD** menu where you can either:

- Change the limits to specify a different interval.
- Change the parameters to find a different root.
- Change only the allowable error to find the root more accurately.

## Example: The Bisection Method

The following example demonstrates how to use the bisection method program.

### Example

Find the root of the function  $y = 3x^2 + 2x - 7$ , which lies between 0 and 20. Use an allowable error of .0001.

Procedure	Press	Display
Enter the function	<b>LEARN</b> <1st> <b>2nd</b> <b>LBL</b> <b>2nd</b> f <b>2nd</b> x ( 3 <b>x</b> <b>RCL</b> A <b>x<sup>2</sup></b> <b>+</b> 2 <b>x</b> <b>RCL</b> A <b>-</b> 7 <b>)</b> <b>2nd</b> <b>RTN</b> <b>LEARN</b>	
Select the program	<b>RUN</b> <MTH> <--> <ZRO> <BIS>	BISECTION METHOD
Enter the lower limit	0 <LO>	LO = 0.
Enter the upper limit	20 <HI>	HI = 20.
Enter the allowable error	.0001 <err>	err = 0.0001
Proceed with result	<EOD>	x = 1.23008728

# Newton's Method

---

This program searches for a root of a function. It converges for most functions, but in some cases goes into oscillation. To prevent the program from running indefinitely, you can specify a limit for the number of iterations.

---

## Introduction

The Newton's method program uses the Newton-Raphson technique to search for a root of a function. For many functions, any initial guess leads quickly to a root. However, an initial guess near the desired root usually shortens execution time.

You must enter the function as a subroutine in program memory before running the program. When the program calls the subroutine, the value for the independent variable is available in both the display and data register A (000).

If the function is a polynomial, you can use the Q-D method to obtain an initial guess.

## Reference

*Handbook of Engineering Fundamentals*, Ovid W. Eshbach, John Wiley & Sons, Inc., New York, 1954, pp. 2-16.

### For the Beginning Programmer

Before using the Newton's method program, you must store the function as a subroutine in program memory.

If you are not familiar with keystroke programming, refer to the following chapters in the *TI-95 Programming Guide* for instructions.

- ▶ "Working with Programs on the TI-95"
- ▶ "Using Calculator Keystrokes in a Program"
- ▶ "Controlling the Sequence of Operations"

### Rules for Storing the Function

When you store the function as a subroutine, follow these rules.

- ▶ The subroutine must be labeled  $fx$  (lowercase only).
- ▶ Anytime the subroutine needs the independent variable, use a RCL A or RCL 000 instruction.
- ▶ The subroutine must terminate with a RTN instruction.



---

## Starting the Program

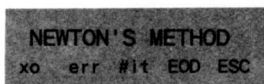
To start the program that uses Newton's method:

1. Store the function as a subroutine, following the rules listed on the previous page.
2. If the subroutine involves any of the trig functions, select the angle units you want to use.
3. Select <ZRO> from the **MATHEMATICS** menu.

The calculator displays the **FUNCTION ZEROS** menu.

4. Press <NTN>.

The program displays:



```
NEWTON'S METHOD
xo  err  #it  EOD  ESC
```

5. Enter the initial guess and press <xo>.
6. Enter the allowable error and press <err>.
7. Enter the maximum number of iterations and press <#it>.

### Finding the Root

After you enter the parameters for the program, you can find the root of the function you have stored.

1. Press <EOD>.

If the program is able to find a root within the number of iterations you specified, it displays:

```
x =          xxx
          ESC
```

where *xxx* is the root

If the program cannot find a root within the specified number of iterations, it displays:

```
#it REACHED
del #it EOD    ESC
```

- To display delta, press <del>.
- To perform additional iterations, enter the number of additional iterations, press <#it>, and press <EOD>. The program continues the calculations for the number of iterations you specify.

## Example: Newton's Method

The following example demonstrates how to use the Newton's method program.

### Example

Find the roots of the function  $y = x^3 - 4x^2 - x + 4$ .  
Use an allowable error of .0001, a limit of 50 iterations,  
and initial guesses of -4, .5, and 10.

Procedure	Press	Display
Enter the function	<b>LEARN</b> <1st> <b>2nd</b> <b>LBL</b> <b>2nd</b> <b>f</b> <b>2nd</b> <b>x</b> <b>(</b> <b>RCL</b> <b>A</b> <b>y<sup>x</sup></b> <b>3</b> <b>-</b> <b>4</b> <b>x</b> <b>RCL</b> <b>A</b> <b>x<sup>2</sup></b> <b>-</b> <b>RCL</b> <b>A</b> <b>+</b> <b>4</b> <b>)</b> <b>2nd</b> <b>RTN</b> <b>LEARN</b>	
Select the program	<b>RUN</b> <MTH> <--> <ZRO> <NTN> <b>NEWTON'S METHOD</b>	
Enter the initial guess	4 <b>[+/-]</b> <x>	xo = -4.
Enter the allowable error	.0001 <err>	err = 0.0001
Enter the number of iterations allowed	50 <#it>	#it = 50.
Proceed with result	<EOD>	x = -1.
Enter the initial guess for a different root	<ESC> .5 <x>	xo = 0.5
Proceed with result using previously entered allowable error and iterations	<EOD>	x = 1.
Enter the initial guess for a different root	<ESC> 10 <x>	xo = 10.
Proceed with result	<EOD>	x = 4.



# Chapter 9: Solving Differential Equations

---

This chapter describes how to use the Runge-Kutta program to solve differential equations.

---

Table of Contents	The Runge-Kutta Program . . . . .	9-2
	Example: Differential Equations . . . . .	9-9

# The Runge-Kutta Program

---

This program solves a system of differential equations. Higher-order equations must be broken down into first-order equations before you can use the program.

---

## Introduction

The Runge-Kutta program uses a fifth-order Runge-Kutta method to solve a system of first-order differential equations of the type  $y' = f(x, y)$ . The program finds approximate solutions at particular points  $x_0, x_1, \dots, x_n$  where the difference between successive  $x$  values is a constant,  $h = x_{n+1} - x_n$ .

You must enter the differential equations as subroutines labeled  $f1, f2, \dots, fn$  (a maximum of nine) before running the program. The value of the independent variable must be recalled from register E (004). The other variables correspond to registers L, M,  $\dots$ , T (011-019).

The program requires you to reduce higher-order differential equations to first-order equations. Rewrite your equations with an extra variable for each order of derivative so that no variable is differentiated more than once.

For example, the third-order equation  $y''' = x + y$  is defined in terms of first-order equations as follows.

$y = y_1$	Substituting gives:
$y' = y_2$	$y_1' = y_2$
$y'' = y_3$	$y_2' = y_3$
$y''' = x + y_1$	$y_3' = x + y_1$

You would describe these equations with the subroutines below.

LBL f1	LBL f2	LBL f3
RCL M	RCL N	(RCL E + RCL L)
RTN	RTN	RTN

---

## Introduction (Continued)

For the simultaneous solution of several differential equations, you can substitute  $y_1 \dots y_n$  for the existing variables in first-order equations and reduce higher-order equations as shown previously. For example, the equations  $z' = e^{-5x}$  and  $y'' = -y' - 2.5y$  are written as follows.

$z = y_1$	Substituting,
$y = y_2$	$y_1' = e^{-5x}$
$y' = y_3$	$y_2' = y_3$
$y'' = -y' - 2.5y$	$y_3' = -y_3 - 2.5 * y_2$

These equations written as subroutines are:

LBL f1	LBL f2	LBL f3
(5 +/- *	RCL N	(RCL N +/-
RCL E) INV LNX	RTN	- 2.5 * RCL M)
RTN		RTN

You must partition the calculator for at least 83 data registers before running the program.

The program prompts you for the following parameters.

- The number of functions in the system of differential equations tells the program which subroutines to include when performing the solution.
- The h value (step size of x's) has an effect on the accuracy of the solution. A smaller h generally yields greater accuracy but slows the solution.
- The starting and ending x values define the boundaries for x.
- The initial value of each derivative provides boundary conditions necessary for the solution.

## Reference

*Numerical Solution of Ordinary Differential Equations*, Lapidus and Seinfeld.

## For the Beginning Programmer

Before running the Runge-Kutta program, you must store the functions in program memory as subroutines.

If you are not familiar with keystroke programming, refer to the following chapters in the *TI-95 Programming Guide* for instructions.

- ▶ “Working with Programs on the TI-95”
- ▶ “Using Calculator Keystrokes in a Program”
- ▶ “Controlling the Sequence of Operations”

## Rules for Storing the Functions

When you store the functions as subroutines, follow these rules.

- ▶ The subroutines must be labeled beginning with *f1* (lowercase *f* only) and continuing through the last function up to a maximum of nine functions.
- ▶ Anytime a subroutine needs the *x* variable, use a RCL E or RCL 004 instruction. The program stores the values of the dependent variables as follows:

Dependent Variable	Register
$y_1$	L (011)
$y_2$	M (012)
$y_3$	N (013)
$y_4$	O (014)
$y_5$	P (015)
$y_6$	Q (016)
$y_7$	R (017)
$y_8$	S (018)
$y_9$	T (019)

- ▶ Each subroutine must terminate with a RTN instruction.



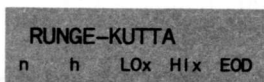
---

## Starting the Program

To start the Runge-Kutta program:

1. Store the functions needed by the program, following the rules on the previous page.
2. If the subroutines involve any of the trig functions, select the angle units you want to use.
3. Select <R-K> from the **MATHEMATICS** menu.

The program displays:

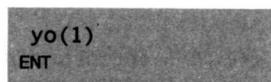


```
RUNGE-KUTTA
n      h      LOx HIx  EOD
```

4. Enter the number of functions stored and press <n>.
5. Enter the step size and press <h>.
6. Enter the initial value of x and press <LOx>.
7. Enter the ending value of x and press <HIx>.
8. Press <EOD>.

## Entering the Initial Values

When you press <EOD>, the program prompts you to enter the initial value of y for each function.



```
y0(1)
ENT
```

Enter all the initial values of y, pressing <ENT> after each value.

### The Edit Menu

After you enter the initial values of  $y$ , the program displays an **EDIT?** menu that allows you to change any of the values.



```
EDIT?  
YES NO
```

- ▶ If you do not want to edit, press <NO> and go to the next page.
- ▶ If you want to edit, press <YES> and use the procedure below.

### Editing the Initial Values

When you select <YES> from the **EDIT?** menu, the program displays the same menu that you used to enter the initial values.



```
yo(1)  
ENT
```

- ▶ To display the current value, press the **CE** key.
- ▶ To accept the current value and proceed to the next  $y$  value, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you press <ENT> to accept or edit the last  $y$  value, the program returns to the **EDIT?** menu.

---

## Choosing the Results

When you select <NO> from the **EDIT?** menu, the program displays:

```
END RESULT ONLY?  
YES NO
```

- ▶ To display the y values at each increment between the low and high values of x, press <NO> and go to the next page.
- ▶ To display the y values at the high value of x only, press <YES> and use the procedure below.

## Displaying End Results Only

When you press <YES>, the program displays:

```
x =          xxx      where xxx is the value  
NXT          ESC
```

1. Press <NXT>.

The program displays the corresponding y values, beginning with the y value for the first function.

```
y1 =          xxx      where xxx is the value of  
NXT          y
```

2. Press <NXT> repeatedly to display the y value for each remaining function.

When the y value for the last function is displayed, pressing <NXT> redisplay the high value of x. (Anytime x is displayed, you can return to the **RUNGE-KUTTA** menu by pressing <ESC>.)

### Displaying Incremental Results

When you select <NO> from the **END RESULT ONLY?** menu, the program displays the first incremental value of  $x$ .

$x =$   
NXT

xxx  
ESC

where  $xxx$  is the value

1. Press <NXT>.

The program displays the corresponding  $y$  values, beginning with the  $y$  value for the first function.

$y1 =$   
NXT

xxx

where  $xxx$  is the value of  
 $y$

2. Press <NXT> repeatedly to display the  $y$  value for each remaining function.

When the  $y$  value for the last function is displayed, pressing <NXT> redisplay the next incremental value of  $x$ .

3. Repeat steps 1 and 2 until all values of  $x$  and  $y$  have been displayed.

When all values of  $y$  have been displayed for the ending value of  $x$ , pressing <NXT> redisplay the first value of  $x$ . (Anytime  $x$  is displayed, you can return to the **RUNGE-KUTTA** menu by pressing <ESC>.)

## Example: Differential Equations

The following example demonstrates how to use the Runge-Kutta program.

### Example

Find the solution for the equation  $Y'' = e^{2x}$  at  $X_f = 1$  with the initial conditions  $X_0 = 0$ ,  $Y(0) = 1$ ,  $Y'(0) = 2$ , and a step size of .2.

This second-order equation reduces to the first-order equations  $y_1' = y_2$  and  $y_2' = e^{2x}$ . The initial conditions become  $y_1(0) = 1$  and  $y_2(0) = 2$ .

Procedure	Press	Display
Enter functions	<b>LEARN</b> <1st> <b>2nd</b> [ <b>LBL</b> ] <b>2nd</b> f1 <b>RCL</b> M <b>2nd</b> [ <b>RTN</b> ] <b>2nd</b> [ <b>LBL</b> ] <b>2nd</b> f2 ( <b>(</b> 2 <b>x</b> <b>RCL</b> E ) <b>INV</b> <b>LN</b> <b>2nd</b> [ <b>RTN</b> ] <b>LEARN</b>	
Select program	<b>RUN</b> <MTH> <--> <R-K> RUNGE-KUTTA	
Enter number of functions	2 <n>	n = 2
Enter the step size	.2 <h>	h = 0.2
Enter initial value of x	0 <LOx>	LOx = 0.
Enter ending value of x	1 <HIx>	HIx = 1.
Proceed with program	<EOD>	yo(1)
Enter the initial y values	1 <ENT>	yo(2)
	2 <ENT>	EDIT?

(continued)

## Example: Differential Equations (Continued)

Example  
(Continued)

Procedure	Press	Display
Proceed with results	<NO>	END RESULT ONLY?
Choose only the end results	<YES>	x = 1.
	<NXT>	y1 = 4.097267833
	<NXT>	y2 = 5.194528357

The solution for  $y''$  is the value of  $y2$ .

# Chapter 10: Number Theory

---

This chapter tells you how to use the Number Theory program.

---

Table of Contents	The Number Theory Program . . . . .	10-2
	The $\phi(n)$ , $\sigma_0$ , and $\sigma_1$ Functions . . . . .	10-4
	Example: $\phi(n)$ , $\sigma_0$ , and $\sigma_1$ . . . . .	10-5
	Congruence Calculations . . . . .	10-6
	Example: Congruence Calculations . . . . .	10-7
	Rational Approximations . . . . .	10-8
	Example: Rational Approximations . . . . .	10-9

# The Number Theory Program

---

Each of the selections in this program treats a number as a combination of integers. The program derives its results from comparisons of the divisors of numbers. Numbers that have no divisors in common are relatively prime.

---

## Introduction

The number theory program has five selections that involve relatively prime numbers, divisors, congruence, and ratios.

The program can determine how many of the integers less than the given integer are relatively prime (have no factors in common with the number). For example, 9 has six relative primes, 1, 2, 4, 5, 7, and 8.

The program can calculate how many numbers are a divisor of a given number. It can also provide the sum of the divisors. For example, the six divisors of 12 are 1, 2, 3, 4, 6, and 12, which total to 28. Outside this program, the calculator can perform other divisor calculations, available from the keyboard.

The program has an option to perform congruence calculations. The congruence of two numbers depends on the modulus you specify. A modulus of 7, for example, causes 12 and 19 to be congruent (both are 5 greater than a multiple of 7).

The standard expression for congruence is  $ax \cong b \pmod{m}$ , which states that for modulus  $m$ ,  $ax$  and  $b$  are congruent. The program finds the values of  $x \leq m$  such that  $\text{frac}((ax - b) \div m) = 0$ . For example, if you enter  $a = 3$ ,  $b = 19$ , and  $m = 7$ , the program results in  $x = 4$ .

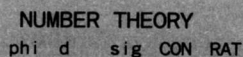
The final option of this program is the search for two integers whose ratio approximates the given value within a specified error. For example,  $3.375$  is equal to  $27/8$ .



---

## The Number Theory Menu

When you select <NUM> from the **MATHEMATICS** menu, the calculator displays:



NUMBER THEORY  
phi d sig CON RAT

The **NUMBER THEORY** menu enables you to select from the following options.

- <phi> Performs the  $((\phi))(n)$  calculation, described on the next page.
- <d> Performs the  $\sigma_0$  calculation, described on the next page.
- <sig> Performs the  $\sigma_1$  calculation, described on the next page.
- <CON> Runs the Congruence program, described on page 10-6.
- <RAT> Runs the Rational Approximation program, described on page 10-8.

## The $((\phi))(n)$ , $\sigma_0$ , and $\sigma_1$ Functions

---

You can calculate  $((\phi))(n)$ ,  $\sigma_0$ , or  $\sigma_1$  directly from the **NUMBER THEORY** menu.

---

### Calculating $((\phi))(n)$ , $\sigma_0$ , or $\sigma_1$

To calculate  $((\phi))(n)$ ,  $\sigma_0$ , or  $\sigma_1$ :

1. Select  $\langle \text{NUM} \rangle$  from the **MATHEMATICS** menu.

The calculator displays the **NUMBER THEORY** menu shown on the previous page.

2. Enter the value of  $n$ .
3. Press the applicable key for the function you want to perform.

$\langle \phi \rangle$       Calculates  $((\phi))(n)$ , the number of integers not exceeding and relatively prime to  $n$ .

$\langle d \rangle$       Calculates  $\sigma_0$ , the number of divisors of  $n$ .

$\langle \text{sig} \rangle$       Calculates  $\sigma_1$ , the sum of the divisors of  $n$ .

The program displays the result.

## Example: The $\phi(n)$ , $\sigma_0$ , and $\sigma_1$ Functions

The following example illustrates the use of the  $\langle \phi \rangle$ ,  $\langle d \rangle$ , and  $\langle \sigma \rangle$  selections from the NUMBER THEORY menu.

### Example

For the number 60, find the number of relative primes, the number of divisors, and the sum of the divisors.

Procedure	Press	Display
Select the program	<b>RUN</b> $\langle \text{MTH} \rangle$ $\langle \text{---} \rangle \langle \text{---} \rangle$ $\langle \text{NUM} \rangle$	NUMBER THEORY
Find the number of relative primes	60 $\langle \phi \rangle$	phi = 16.
Find the number of divisors	60 $\langle d \rangle$	d = 12.
Find the sum of the divisors	60 $\langle \sigma \rangle$	sig = 168.

## Congruence Calculations

---

The <CON> selection from the **NUMBER THEORY** menu lets you perform congruence calculations. Given  $a$ ,  $b$ , and  $m$ , the program searches for all values of  $x$  such that  $ax$  is congruent to  $b$  for modulus  $m$ .

---

### Performing Congruence Calculations

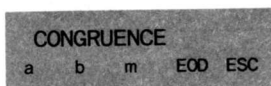
To perform a congruence calculation:

1. Select <NUM> from the **MATHEMATICS** menu.

The calculator displays the **NUMBER THEORY** menu.

2. Select <CON> from the menu.

The calculator displays:



3. Enter the values of  $a$ ,  $b$ , and  $m$ , pressing the applicable key after entering each value.
4. Press <EOD> to continue with the program and perform the calculation.
  - ▶ If there is no solution, the message **NO SOLUTION** is displayed.
  - ▶ If there are solutions, the value of  $x$  for the first solution is displayed.



where  $xxx$  is the solution

5. Press <NXT> repeatedly to display the remaining solutions for which  $x \leq m$ .

When all the solutions have been displayed, pressing <NXT> returns you to the **CONGRUENCE** menu.

## Example: Congruence Calculations

The following example illustrates the use of the <CON> selection from the NUMBER THEORY menu.

### Example

Solve for multipliers of 4 that cause congruence with 4 for a modulus of 8. Also, solve for multipliers of 5 that cause congruence with 2 for a modulus of 5.

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <--> <NUM> <CON>	CONGRUENCE
Enter a	4 <a>	a = 4.
Enter b	4 <b>	b = 4.
Enter m	8 <m>	m = 8.
Proceed with results	<EOD>	x 1 = 1.
	<NXT>	x 2 = 3.
	<NXT>	x 3 = 5.
	<NXT>	x 4 = 7.
	<NXT>	CONGRUENCE
Enter a	5 <a>	a = 5.
Enter b	2 <b>	b = 2.
Enter m	5 <m>	m = 5.
Proceed with results	<EOD>	NO SOLUTION

$4 \cdot 1$ ,  $4 \cdot 3$ ,  $4 \cdot 5$ , and  $4 \cdot 7$  are all congruent to 4 for a modulus of 8. However, 5 has no multiplier that causes congruence with 2 for a modulus of 5.

## Rational Approximations

---

The <RAT> selection from the **NUMBER THEORY** menu lets you calculate a fraction that approximates a specified number, within a given error.

---

### Calculating a Rational Approximation

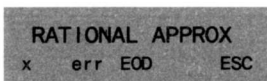
To calculate the rational approximation of a number:

1. Select <NUM> from the **MATHEMATICS** menu.

The calculator displays the **NUMBER THEORY** menu.

2. Select <RAT> from the menu.

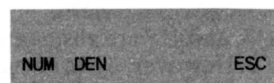
The calculator displays:



RATIONAL APPROX  
x err EOD ESC

3. Enter the number you want to approximate and press <x>. (This number is not retained when the result is calculated, so you must enter a value for x each time this menu appears.)
4. Enter the allowable error and press <err>. (If you enter 0, the result is within the limits of the calculator.)
5. Press <EOD> to continue with the program and calculate the fraction.

When the approximation is complete, the calculator displays:



NUM DEN ESC

6. Press <NUM> to display the numerator.
7. Press <DEN> to display the denominator.

## Example: Rational Approximations

The following example illustrates the use of the <RAT> selection from the NUMBER THEORY menu.

### Example

Determine rational approximations with an accuracy of .000001 for the numbers 1.75,  $\sqrt{2}$ , and  $\log(e)$ .

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <--> <NUM> <RAT>	RATIONAL APPROX
Enter the number	1.75 <x>	x = 1.75
Enter the allowable error	.000001 <err>	err = 0.000001
Proceed with results	<EOD>	0.
View the numerator	<NUM>	NUM = 7.
View the denominator	<DEN>	DEN = 4.
Enter the next number	<ESC> 2 <b>√x</b> <x>	x = 1.414213562
Proceed with results using the same error	<EOD>	0.
View the numerator	<NUM>	NUM = 1393.
View the denominator	<DEN>	DEN = 985.
Enter the next number	<ESC> 1 <b>INV</b> <b>LN</b> <b>log</b> <x>	x = .4342944819
Proceed with results using the same error	<EOD>	0.
View the numerator	<NUM>	NUM = 271.
View the denominator	<DEN>	DEN = 624.





# Chapter 11: Transforming Coordinates

---

This chapter describes how to use the Coordinate Transforms program.

---

<b>Table of Contents</b>	<b>The Coordinate Transforms Program . . . . .</b>	<b>11-2</b>
	<b>Example: Coordinate Transforms . . . . .</b>	<b>11-6</b>

# The Coordinate Transforms Program

---

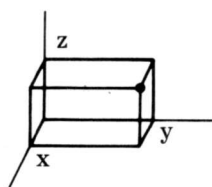
This program lets you transform three-dimensional coordinates between the rectangular, cylindrical, and spherical coordinate systems.

---

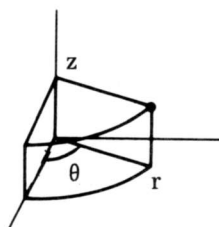
## Introduction

The Coordinate Transforms program is designed to accept coordinates in any of three coordinate systems and convert them to any of the other coordinate systems. All angles are interpreted according to the angle units setting of the calculator.

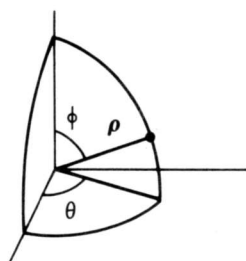
### Rectangular Coordinate System, Right-hand Axes



### Cylindrical Coordinates



### Spherical Coordinates



---

## Starting the Program

To start the Coordinate Transforms program, select  $\langle XFM \rangle$  from the **MATHEMATICS** menu.

The program displays:

```
COORD TRANSFORMS
R-C  R-S  C-S
```

$\langle R-C \rangle$  Transforms rectangular to cylindrical.

**INV**  $\langle R-C \rangle$  Transforms cylindrical to rectangular.

$\langle R-S \rangle$  Transforms rectangular to spherical.

**INV**  $\langle R-S \rangle$  Transforms spherical to rectangular.

$\langle C-S \rangle$  Transforms cylindrical to spherical.

**INV**  $\langle C-S \rangle$  Transforms spherical to cylindrical.

### Entering the Coordinates

After displaying the **COORD TRANSFORMS** menu, use the following instructions to enter the original coordinates.

1. Select the angle units you want to use.
2. Select the type of transformation you want to perform.

The program prompts you to enter the original coordinates.

For example, if you press <R-C> the program displays:

```
RECT->CYL
x   y   z   EOD  ESC
```

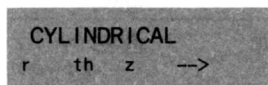
3. Enter the x-, y-, and z-coordinates, pressing the applicable key after entering each value.
4. Press <EOD> to indicate you have finished entering the values.

---

## Displaying the Results

When you press <EOD>, the program displays a menu that lets you display the transformed coordinates.

For example, if you are transforming values to cylindrical coordinates, the menu lets you display the radius, theta, and the z-coordinate.



```
CYLINDRICAL
r   th  z   -->
```

1. Press the applicable key for the coordinate you want to display.

The program displays the value of the selected coordinate.

2. When you finish displaying the transformed coordinates, press <--> to return to the input menu for the same type of transformation.

## Example: Coordinate Transforms

The following example illustrates the use of the Coordinate Transforms program.

### Example

Transform the rectangular coordinates (2.5, 4, 3) to cylindrical and spherical coordinates.

Procedure	Press	Display
Set angle units to degrees	<b>INV</b> <b>2ND</b> <b>DRG</b>	DEG MODE
Select program	<b>RUN</b> <MTH> <--> <--> <XFM>	COORD TRANSFORMS
Select transform type	<R-C>	RECT ->CYL
Enter coordinates	2.5 <x>	x = 2.5
	4 <y>	y = 4.
	3 <z>	z = 3.
Proceed with results	<EOD>	CYLINDRICAL
View cylindrical coordinates	<r>	r = 4.716990566
	<th>	th = 57.99461679
	<z>	z = 3.
Proceed with program	<--> <ESC>	COORD TRANSFORMS
Select transform type	<C-S>	CLY ->SPHERE
Proceed with results using current values	<EOD>	SPHERICAL
View spherical coordinates	<rho>	rho = 5.590169944
	<th>	th = 57.99461679
	<phi>	phi = 57.54369154

# Chapter 12: Analytic Geometry

---

This chapter describes how to use the programs available through the <GEO> selection of the MATHEMATICS menu.

---

<b>Table of Contents</b>	<b>Introduction</b> .....	<b>12-2</b>
	<b>The Conic Sections Program</b> .....	<b>12-3</b>
	<b>Example: Conic Sections</b> .....	<b>12-14</b>
	<b>The Quadric Surfaces Program</b> .....	<b>12-16</b>
	<b>Example: Quadric Surfaces</b> .....	<b>12-22</b>

# Introduction

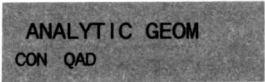
---

Two programs are available from the ANALYTIC GEOM menu.

---

## The Analytic Geometry Menu

When you select <GEO> from the **MATHEMATICS** menu, the calculator displays:



ANALYTIC GEOM  
CON QAD

- <CON>      Runs the Conic Sections program, described on the next page.
- <QAD>      Runs the Quadric Surfaces program, described on page 12-16.



# The Conic Sections Program

---

This program identifies any conic section from the coefficients of the general second-degree equation in two variables.

$$Ax^2 + Hxy + By^2 + Gx + Fy + C = 0$$

---

## Introduction

The Conic Sections program analyzes the six coefficients of the general second-degree equation to identify the type of conic section. The possibilities are:

- ▶ **Ellipse (single point in real plane)**—The program identifies the conic section as a point ellipse and supplies the coordinates of the point.
- ▶ **Ellipse (in real plane)**—The program supplies the rotation angle that eliminates the  $xy$  term, the coordinates of the center for the original axes and rotated axes, the lengths of the major and minor axes, and the equation of the principal axis.
- ▶ **Ellipse (no points in real plane)**—The program identifies the conic section as an imaginary ellipse.
- ▶ **Hyperbola**—The program supplies the rotation angle that eliminates the  $xy$  term, the coordinates of the center for the original axes and rotated axes, the lengths of the transverse and conjugate axes, and the equation of the principal axis.
- ▶ **Parabola**—The program supplies the rotation angle that eliminates the  $xy$  term, the coordinates of the vertex relative to the original and rotated axes, the length and sign of the latus rectum, and the equation of the axis of symmetry.
- ▶ **Intersecting or parallel lines (in real plane)**—The program supplies the equations of the lines.
- ▶ **Parallel lines (no points in real plane)**—The program identifies the conic section as complex conjugate lines.

Before using the program, make sure the calculator is partitioned for at least 29 data registers.

### Starting the Program

To start the Conic Sections program:

1. Select <GEO> from the **MATHEMATICS** menu.

The calculator displays the **ANALYTIC GEOM** menu.

2. Select <CON>.

The program displays:

```
CONIC SECTIONS
A  H  B  --> ESC
```

### Entering the Coefficients

To enter the coefficients for the general quadratic equation:

1. Enter the values of A, H, and B, pressing the applicable key after entering each value.
2. Press <-->.

The program displays:

```
G  F  C  --> EOD
```

3. Enter the values of G, F, and C, pressing the applicable key after entering each value.

If any of the first three coefficients were entered incorrectly, press <--> and re-enter the coefficients.

4. When the coefficients are correct, press <EOD>.

---

**Determining  
the Type of  
Conic Section**

When you press <EOD>, the program displays a message to identify the type of conic section.

Proceed to the page listed below for the message shown in your display.

Message Displayed	Page
POINT ELLIPSE	12-6
REAL ELLIPSE	12-6
IMAG ELLIPSE	12-8
HYPERBOLA	12-8
PARABOLA	12-10
PARALLEL LINES	12-12
INTERSECT LINES	12-13
CMPLX CONJ LINES	12-13

### Point Ellipse

If your equation describes a point ellipse, pressing <EOD> displays:

```
POINT ELLIPSE
x      y      ESC
```

<x> Displays the x-coordinate of the point.

<y> Displays the y-coordinate of the point.

<ESC> Returns to the **CONIC SECTIONS** menu.

### Real Ellipse

If your equation describes a real ellipse, pressing <EOD> displays:

```
REAL ELLIPSE
-->      ESC
```

1. Press <-->.

The program displays:

```
ORIGINAL CENTER
x      y      -->
```

<x> Displays the x-coordinate for the center of the original ellipse.

<y> Displays the y-coordinate for the center of the original ellipse.

---

**Real Ellipse  
(Continued)**

2. To continue, press <-->.

The program displays:

```
ROTATED CENTER
ang  x'  y'  -->
```

- <ang> Displays the angle of rotation.
- <x'> Displays the x-coordinate of the center with respect to the rotated axes.
- <y'> Displays the y-coordinate of the center with respect to the rotated axes.

3. To continue, press <-->.

The program displays:

```
AXES
maj min m b -->
```

- <maj> Displays the length of the major axis.
- <min> Displays the length of the minor axis.
- <m> Displays the slope of the major axis with respect to the original axes.
- <b> Displays the intercept of the major axis with respect to the original axes.
4. When you finish viewing the results, press <--> to return to the **REAL ELLIPSE** menu.
5. Press <ESC> to return to the **CONIC SECTIONS** menu.

### Imaginary Ellipse

If your equation describes an imaginary ellipse, pressing <EOD> displays:

```
IMAG ELLIPSE
                               ESC
```

Press <ESC> to return to the **CONIC SECTIONS** menu.

### Hyperbola

If your equation describes a hyperbola, pressing <EOD> displays:

```
HYPERBOLA
-->                               ESC
```

1. Press <-->.

The program displays:

```
ORIGINAL CENTER
x      y  -->
```

- <x>      Displays the x-coordinate for the center of the original ellipse.
- <y>      Displays the y-coordinate for the center of the original ellipse.

---

**Hyperbola**  
**(Continued)**

2. To continue, press <-->.

The program displays:

```
ROTATED CENTER
ang x' y' -->
```

- <ang> Displays the angle of rotation.
- <x'> Displays the x-coordinate of the center with respect to the rotated axes.
- <y'> Displays the y-coordinate of the center with respect to the rotated axes.

3. To continue, press <-->.

The program displays:

```
AXES
trn cnj m b -->
```

- <trn> Displays the length of the transverse axis.
- <cnj> Displays the length of the conjugate axis.
- <m> Displays the slope of the transverse axis with respect to the original axes.
- <b> Displays the intercept of the transverse axis with respect to the original axes.

4. When you finish viewing the results, press <--> to return to the **HYPERBOLA** menu.
5. Press <ESC> to return to the **CONIC SECTIONS** menu.

## Parabola

If your equation describes a parabola, pressing <EOD> displays:

```
PARABOLA
-->                               ESC
```

1. Press <-->.

The program displays:

```
ORIGINAL VERTEX
x   y   -->
```

- <x> Displays the x-coordinate for the vertex of the original parabola.
- <y> Displays the y-coordinate for the vertex of the original parabola.

2. To continue, press <-->.

The program displays:

```
ROTATED VERTEX
ang x' y' -->
```

- <ang> Displays the angle of rotation.
- <x'> Displays the x-coordinate of the vertex with respect to the rotated axes.
- <y'> Displays the y-coordinate of the vertex with respect to the rotated axes.



---

Parabola  
(Continued)

3. To continue, press <-->.

The program displays:

```
AXES
LR  m  b  -->
```

- <LR> Displays the length of the latus rectum. (If the sign is positive, the parabola opens to the left. If it is negative, the parabola opens to the right.)
- <m> Displays the slope of the latus rectum with respect to the original axes.
- <b> Displays the intercept of the latus rectum with respect to the original axes.
4. When you finish viewing the results, press <--> to return to the **PARABOLA** menu.
5. Press <ESC> to return to the **CONIC SECTIONS** menu.

### Parallel Lines

If your equation describes parallel lines, pressing <EOD> displays:

```
PARALLEL LINES
-->                               ESC
```

1. Press <-->.

The program displays:

```
EQUATIONS
m1  b1  m2  b2  -->
```

- |      |   |
|------|---|
| <m1> | Displays the slope of the first line.     |
| <b1> | Displays the intercept of the first line. |
| <m2> | Displays the slope of the other line.     |
| <b2> | Displays the intercept of the other line. |
2. When you finish viewing the results, press <--> to return to the **PARALLEL LINES** menu.
  3. Press <ESC> to return to the **CONIC SECTIONS** menu.

---

## Intersecting Lines

If your equation describes intersecting lines, pressing  $\langle \text{EOD} \rangle$  displays:

```
INTERSECT LINES
-->                      ESC
```

1. Press  $\langle --> \rangle$ .

The program displays:

```
EQUATIONS
m1  b1  m2  b2  -->
```

- $\langle m1 \rangle$  Displays the slope of the first line.
- $\langle b1 \rangle$  Displays the intercept of the first line.
- $\langle m2 \rangle$  Displays the slope of the second line.
- $\langle b2 \rangle$  Displays the intercept of the second line.

2. When you finish viewing the results, press  $\langle --> \rangle$  to return to the **INTERSECT LINES** menu.
3. Press  $\langle \text{ESC} \rangle$  to return to the **CONIC SECTIONS** menu.

## Complex Conjugate Lines

If your equation describes a pair of complex conjugate lines, pressing  $\langle \text{EOD} \rangle$  displays:

```
CMPLX CONJ LINES
                      ESC
```

Press  $\langle \text{ESC} \rangle$  to return to the **CONIC SECTIONS** menu.

## Example: Conic Sections

The following example illustrates the use of the Conic Sections program.

### Example

Determine the nature of the curve described by the equation

$$44x^2 + 600xy - 551y^2 + 1976x - 2106y - 1859 = 0$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <--> <GEO> <CON>	CONIC SECTIONS
Enter the coefficients	44 <A>	A = 44.
	600 <H>	H = 600.
	551 <b>+/-</b> <B>	B = -551.
	<--> 1976 <G>	G = 1976.
	2106 <b>+/-</b> <F>	F = -2106.
	1859 <b>+/-</b> <C>	C = -1859.
Proceed with program	<EOD>	HYPERBOLA
View original center coordinates	<-->	ORIGINAL CENTER
	<x>	x = -2.
	<y>	y = -3.

**Example  
(Continued)**

Procedure	Press	Display
View rotated center parameters	$\langle -- \rangle$	ROTATED CENTER
	$\langle \text{ang} \rangle$	ang = .3947911197
	$\langle x' \rangle$	x' = -3.
	$\langle y' \rangle$	y' = -2.
View parameters of axes	$\langle -- \rangle$	AXES
	$\langle \text{trn} \rangle$	trn = 4.
	$\langle \text{cnj} \rangle$	cnj = 2.
	$\langle m \rangle$	m = .4166666667
	$\langle b \rangle$	b = -2.166666667

# The Quadric Surfaces Program

---

This program identifies any quadric surface from the coefficients of the general second-degree equation in three variables.

$$Ax^2 + By^2 + Cz^2 + Fyz + Gxz + Hxy + Px + Qy + Rz + D = 0$$

---

## Introduction

The Quadric Surfaces program analyzes the ten coefficients of the general second-degree equation to identify the type of quadric surface. At least one of the A, B, C, F, G, or H coefficients must be nonzero. The program rotates the coordinate axes to eliminate any cross-product terms. The possibilities for the surface are:

- ▶ Elliptical or hyperbolic paraboloid—The program supplies the coordinates of the critical point with respect to the rotated axes.
- ▶ Real or imaginary ellipsoid—The program supplies the coordinates of the center with respect to the rotated axes.
- ▶ Hyperboloid of one or two sheets—The program supplies the coordinates of the center with respect to the rotated axes.
- ▶ Cylinder—The program determines the variable to be eliminated from the original equation.
- ▶ Two planes—The program determines the variable to be eliminated from the original equation.
- ▶ Imaginary cone—The program supplies the coordinates of the center with respect to the rotated axes.

Before using the program, make sure the calculator is partitioned for at least 33 data registers.

After using the Quadric Surfaces program to provide simplified coefficients, you can use the Conic Sections program to help determine the exact nature of a surface.

---

## Starting the Program

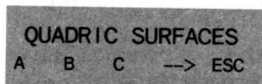
To start the Quadric Surfaces program:

1. Select <GEO> from the **MATHEMATICS** menu.

The calculator displays the **ANALYTIC GEOM** menu.

2. Press <QAD>.

The program displays:



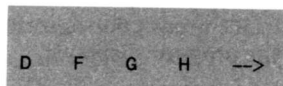
QUADRIC SURFACES  
A B C --> ESC

## Entering the Coefficients

To enter the coefficients of the three-dimensional quadratic equation:

1. Enter the values of A, B, and C, pressing the applicable key after entering each value.
2. Press <-->.

The program displays:



D F G H -->

3. Enter the values of D, F, G, and H, pressing the applicable key after entering each value.

(continued)

### Entering the Coefficients (Continued)

4. Press <-->.

The program displays:

```
P   Q   R   --> EOD
```

5. Enter the values of P, Q, and R, pressing the applicable key after entering each value.

If any of the coefficients of an earlier group were entered incorrectly, press <--> and re-enter the coefficients.

6. When the coefficients are correct, press <EOD>.

### Determining the Type of Surface

When you press <EOD>, the program displays a message to identify the type of quadric surface.

The message will be one of those listed on the next page, accompanied by a <--> selection to let you display data about the surface.

For example, if the equation describes a hyperboloid of one sheet, the program displays:

```
1-S HYPERBOLOID
```

```
-->
```



---

Determining  
the Type of  
Surface  
(Continued)

Message Displayed	Type of Surface
ELLIP PARABOLOID	Elliptical paraboloid
HYPER PARABOLOID	Hyperbolic paraboloid
REAL ELLIPSOID	Real ellipsoid
IMAG ELLIPSOID	Imaginary ellipsoid
1_S HYPERBOLOID	Hyperboloid of one sheet
2_S HYPERBOLOID	Hyperboloid of two sheets
CYL OR 2 PLANES	Cylinder or two planes
IMAGINARY CONE	Imaginary cone

### Displaying the Rotated Coefficients

When the axes are rotated to eliminate the cross-product terms, the general equation becomes:

$$Ax^2 + By^2 + Cz^2 + Px + Qy + Rz + D = 0$$

To display the coefficients for this equation when the type of surface is displayed:

1. Press <-->.

The program displays:

```
ROTATED COEFF
A   B   C   D   -->
```

2. Press the applicable key to display the A, B, C, or D coefficient.
3. When you are ready to proceed, press <-->.

The program displays:

```
P   Q   R   --> ESC
```

4. Press the applicable key to display the P, Q, or R coefficient.

If you want to redisplay A, B, C, or D, press <-->.

---

### Displaying the Center or the Vertex

To display the location of the center or the vertex of the surface:

1. Press  $\langle \text{ESC} \rangle$  after viewing the rotated coefficients.

The program displays one of two menus, depending on the type of surface.

```
CENTER LOCATION
x   y   z       ESC
```

```
VERTEX LOCATION
x   y   z       ESC
```

2. Press the applicable key to display the x-, y-, or z-coordinate.
3. When you finish viewing the coordinates, press  $\langle \text{ESC} \rangle$  to return to the **QUADRIC SURFACES** menu.

## Example: Quadric Surfaces

The following example illustrates the use of the Quadric Surfaces program.

### Example

Determine the nature of the surface described by the equation

$$7x^2 + 7y^2 + 10z^2 + 4yz - 4xz - 2xy - 12x + 12y + 60z - 24 = 0$$

Procedure	Press	Display
Select the program	<b>RUN</b> <MTH> <--> <--> <GEO> <QAD>	QUADRIC SURFACES
Enter the coefficients	7 <A>	A = 7.
	7 <B>	B = 7.
	10 <C>	C = 10.
	<--> 24 <b>+/-</b> <D>	D = -24.
	4 <F>	F = 4.
	4 <b>+/-</b> <G>	G = -4.
	2 <b>+/-</b> <H>	H = -2.
	<--> 12 <b>+/-</b> <P>	P = -12.
	12 <Q>	Q = 12.
	60 <R>	R = 60.
Proceed with program	<EOD>	REAL ELLIPSOID

Example  
(Continued)

Procedure	Press	Display
View rotated coefficients	$\langle -- \rangle$	ROTATED COEFF
	$\langle A \rangle$	A = 6.
	$\langle B \rangle$	B = 6.
	$\langle C \rangle$	C = 12.
	$\langle D \rangle$	D = - 24.
	$\langle -- \rangle \langle P \rangle$	P = 16.09968944
	$\langle Q \rangle$	Q = - 13.14534138
	$\langle R \rangle$	R = 58.78775383
View center location	$\langle \text{ESC} \rangle$	CENTER LOCATION
	$\langle x \rangle$	x = - 1.341640787
	$\langle y \rangle$	y = 1.095445115
	$\langle z \rangle$	z = - 2.449489743



# Chapter 13: Solving Nonlinear Systems

---

This chapter describes how to use the Nonlinear Systems program.

---

<b>Table of Contents</b>	<b>The Nonlinear Systems Program .....</b>	<b>13-2</b>
	<b>Example: Nonlinear Systems .....</b>	<b>13-8</b>

# The Nonlinear Systems Program

---

**This program solves up to eight simultaneous equations that can be nonlinear functions. It uses functions you store in program memory.**

---

## Introduction

The Nonlinear Systems program approximates the roots of a system of equations using Newton's method. You can enter functions such as higher-order polynomials, trigonometric functions, or linear functions for simultaneous solution. The solution is a point that occurs at the intersection of all the entered functions.

You can enter from two to eight functions to be solved simultaneously. You must enter the functions as subroutines in program memory before running the program. The values of the variables correspond to registers B, C, . . . , I (001-008).

You must partition the calculator for at least 115 data registers before running the program.

The program prompts you for allowable error, maximum number of iterations, initial values for each variable, and an initial guess for the root of each variable.

## Important

The program may not arrive at a solution when the initial guess is too inaccurate. If your guesses cause the **NO SOLUTION** message to appear, repeatedly try the program with different guesses before deciding no solution exists.



---

## For the Beginning Programmer

Before running the Nonlinear Systems program, you must store the functions as subroutines in program memory.

If you are not familiar with keystroke programming, refer to the following chapters in the *TI-95 Programming Guide* for instructions.

- ▶ “Working with Programs on the TI-95”
- ▶ “Using Calculator Keystrokes in a Program”
- ▶ “Controlling the Sequence of Operations”

## Rules for Storing the Functions

When you store the functions as subroutines, follow these rules.

- ▶ The subroutines must be labeled beginning with *f1* (lowercase *f* only), and continuing through the last function up to a maximum of eight functions.
- ▶ The values of the variables must be recalled from the following registers.

Variable	Register
$x_1$	B (001)
$x_2$	C (002)
$x_3$	D (003)
$x_4$	E (004)
$x_5$	F (005)
$x_6$	G (006)
$x_7$	H (007)
$x_8$	I (008)

- ▶ The subroutine for each function must terminate with a RTN instruction.

### Starting the Program

To start the Nonlinear Systems program:

1. Store the functions needed by the program, following the rules on the previous page.
2. If the subroutines involve any of the trig functions, select the angle units you want to use.
3. Select <NON> from the **MATHEMATICS** menu.

The program displays:

```
NONLINEAR SYSTEM
n  err #it EOD
```

4. Enter the number of functions stored and press <n>.
5. Enter the amount of allowable error and press <err>.
6. Enter the number of iterations you want the program to perform and press <#it>.
7. Press <EOD> to indicate the data is complete.

### Entering Initial Guesses

When you press <EOD>, the program prompts you to enter an initial guess for each variable.

```
xo(1)
ENT
```

Enter the values of the guesses, pressing <ENT> after each value.

---

## The Edit Menu

After you enter the initial guesses, the program lets you change any values entered incorrectly.



```
EDIT?  
YES NO
```

- ▶ If you have no changes, press <NO> and proceed to the next page.
- ▶ If you need to change any values, press <YES> and use the procedure described below.

## Editing the Initial Guesses

When you select <YES> from the **EDIT?** menu, the program displays the same menu you used to enter the initial guesses.



```
xo(1)  
ENT
```

- ▶ To display the current value, press the **CE** key.
- ▶ To accept the current value and proceed to the next, press <ENT>.
- ▶ To edit the current value, enter the correct value and press <ENT>.

When you finish editing the initial guesses, press <NO> from the **EDIT?** menu.

### Displaying the Solutions

If the message **NO SOLUTION** or **## REACHED** is displayed, proceed to the next page for instructions.

If the program can find solutions within the number of iterations you specified, it displays the value of the first variable.

x1=	xxx	where xxx is the value
NXT	ESC	

1. Press <NXT> repeatedly to display the solution for each of the remaining variables.

When all the solutions have been displayed, pressing <NXT> redisplay the value for the first variable.

2. When you finish viewing the solutions, press <ESC>.

The program returns to the **NONLINEAR SYSTEM** menu.

---

**If No Solution  
Exists**

If no solution exists for the system, the program displays the following menu after you press <NO> from the **EDIT?** menu.

```
NO SOLUTION
                                ESC
```

Press <ESC> to return to the **NONLINEAR SYSTEM** menu.

**If the Number  
of Iterations  
Is Reached**

If the program fails to find a solution within the number of iterations you specified, it displays:

```
#it REACHED
del #it EOD      ESC
```

- ▶ To display delta, press <del>.
- ▶ To perform additional iterations, enter the number of additional iterations, press <#it>, and press <EOD>.

The program continues the calculations for the number of iterations you specify.

- ▶ To return to the **NONLINEAR SYSTEM** menu, press <ESC>.

## Example: Nonlinear Systems

The example below demonstrates how to use the Nonlinear Systems program.

**Example** Find the intersection of the following functions with an allowable error of .0001 and a limit of 25 iterations.

$$(x-1)^2 + (y-1)^2 + (z-1)^2 - 6 = 0$$

$$(x+1)^2 + (y+1)^2 + (z+1)^2 - 6 = 0$$

$$yz^2 + x^3 = 0$$

Let  $x = x1$ ,  $y = x2$ , and  $z = x3$ .

Procedure	Press	Display
Enter the functions (The program counter should equal 0088 when you have entered all the functions.)	<b>LEARN</b> <1st> <b>2nd</b> <b>LBL</b> <b>2nd</b> f1 ( ( <b>RCL</b> 001 <b>-</b> 1 ) <b>x</b> <sup>2</sup> <b>+</b> ( <b>RCL</b> 002 <b>-</b> 1 ) <b>x</b> <sup>2</sup> <b>+</b> ( <b>RCL</b> 003 <b>-</b> 1 ) <b>x</b> <sup>2</sup> <b>-</b> 6 ) <b>2nd</b> <b>RTN</b> <b>2nd</b> <b>LBL</b> <b>2nd</b> f2 ( ( <b>RCL</b> 001 <b>+</b> 1 ) <b>x</b> <sup>2</sup> <b>+</b> ( <b>RCL</b> 002 <b>+</b> 1 ) <b>x</b> <sup>2</sup> <b>+</b> ( <b>RCL</b> 003 <b>+</b> 1 ) <b>x</b> <sup>2</sup> <b>-</b> 6 ) <b>2nd</b> <b>RTN</b> <b>2nd</b> <b>LBL</b> <b>2nd</b> f3 ( <b>RCL</b> 002 <b>x</b> <b>RCL</b> 003 <b>x</b> <sup>2</sup> <b>+</b> <b>RCL</b> 001 <b>y</b> <sup>x</sup> 3 ) <b>2nd</b> <b>RTN</b> <b>LEARN</b>	

**Example  
(Continued)**

Procedure	Press	Display
Select program	<b>RUN</b> <MTH> <--> <--> <NON>	NONLINEAR SYSTEM
Enter the number of functions	3 <n>	n = 3.
Enter the allowable error	.0001 <err>	err = 0.0001
Enter the iteration limit	25 <#it>	#it = 25.
Proceed with program	<EOD>	xo(1)
Enter initial x1	1 <b>+/-</b> <ENT>	xo(2)
Enter initial x2	1 <ENT>	xo(3)
Enter initial x3	0 <ENT>	EDIT?
Proceed with program	<NO>	x1 = - .8032680521
	<NXT>	x2 = 1.409637165
	<NXT>	x3 = - .6063691125





# Appendix A: Registers and Flags Used

---

The register contents provided in this chapter are important when you need to save and retrieve data. The list of flags used in each program is important during programming and debugging.

---

<b>Table of Contents</b>	<b>Register Contents .....</b>	<b>A-2</b>
	<b>Flags Used .....</b>	<b>A-10</b>

# Register Contents

The following table lists the contents of the data registers used by each program. Using these tables, you can determine which data register is occupied by any value a program generates. You also can determine the block of registers to save as a file for those programs that let you use saved data.

	Program Name	Register	Contents
Complex Functions	Complex Functions	t-register	used
		000	real x
		001	imaginary x
		002	real y
		003	imaginary y
		004	used
		005	used
Interpolation	Cubic Splines	t-register	used
		000	available
		001	used
		:	:
		004	used
		005	available
		:	:
		009	available
		010	used
		011	used
		:	:
		6m + 3	used
	Exact Polynomials	t-register	used
		000	used
		:	:
		004	used
		005	available
		006	available
		007	available
		008	x
		009	f(x)
		010	n
		011	x <sub>1</sub>
		:	:
		031	x <sub>max</sub>
		032	y <sub>1</sub>
		:	:
		052	y <sub>max</sub>
		053	c <sub>0</sub>
		:	:
		073	c <sub>max</sub>

---

---

	Program Name	Register	Contents
Gamma Function	Gamma Function	000	x
		001	x
		002	used
		003	used
		004	ln gamma(x)
		005	used
		006	used
Gauss Quadrature	Gauss Quadrature	000	LO
		001	HI
		002	n
		003	used
		:	:
		006	used
		007	x
		008	used
		:	:
Matrix Solutions	Matrix Product	000	m (# of rows in a)
		001	n (# of columns in a)
		002	p (# of columns in b)
		003	used
		:	:
		9 + (m + 1)(n + 1)	used

---

(Continued)

## Register Contents (Continued)

	Program Name	Register	Contents
Matrix Solutions (Continued)	Inversion/ Linear Systems	000	available
		001	used
		:	:
		005	used
		006	A
		007	used
		008	used
		009	status: 0 for none, 1 for factor, 2 for inversion
		010	n (matrix order)
		011	$a_{11}$
		012	$a_{21}$
		:	:
		$10+n^2$	$a_{nn}$
		$11+n^2$	$t_1$ (table)
		:	:
		$10+n^2+n$	$t_n$
		$11+n^2+n$	$b_1 \sim x_1$
		:	:(constant~solution)
		$10+n^2+2n$	$b_n \sim x_n$
		$11+n^2+2n$	$\beta_1$
		:	:(The $\beta$ 's are the b's permuted according to the t's such that $A^{-1}*\beta = x$ to produce the solution.)
		$10+n^2+3n$	$\beta_n$
	Tridiagonal	000	used
		:	:
		009	used
		010	n (order of system)
		011	used
		:	:
		$4n+10$	used

(Continued)

	Program Name	Register	Contents
Matrix Solutions (Continued)	Eigenvalues	000	n (matrix order)
		001	used
		:	:
		015	used
		016	available
		017	available
		018	used
		:	:
		023	used
		024	available
		025	used
		:	:
		$25 + 2n$	used
		$26 + 2n$	$H(1,1)$
		:	:
		$24 + (n+1)^2$	$H(n,n)$
		$25 + (n+1)^2$	$R(1,1)$
		:	:
		$24 + (n+1)^2 + n^2$	$R(n,n)$
		$25 + (n+1)^2 + n^2$	used
Polynomial Product	Polynomial Product	000	degree $P_A$
		001	degree $P_B$
		002	used
		003	used
		004	available
		:	:
		007	available
		008	$a_n$
		:	:
		103	$a_0$
		104	$b_n$
		:	:
		124	$b_0$

## Register Contents (Continued)

	Program Name	Register	Contents
Function Zeros	Q-D and Bairstow	000	used
		:	:
		003	used
		004	r
		005	s
		006	delta
		007	1st root
		008	2nd root
		009	G if Bairstow
		010	#it if Q-D
		011	used
		012	used
		013	#it if Bairstow
		014	used
		:	:
		017	used
		018	n
		019	used
		020	A(0)
		:	:
		045	A(i)
		046	Q(0,0)
		:	:
		071	Q(0,i)
		072	Q(1,0)
		:	:
		097	Q(1,i)
		098	Q(2,0)
		:	:
		123	Q(2,i)
	Bisection	000	x
		001	LO
		002	used
		003	HI
		004	used
		009	error limit

(continued)

	Program Name	Register	Contents
Function Zeros (Continued)	Newton	000	x
		001	used
		005	used
		006	delta
		009	error limit
		010	iteration limit
Runge-Kutta	Runge-Kutta	000	number of functions
		001	stepsize
		002	initial x and step x
		003	final x
		004	evaluation x
		005	used
		:	:
		010	used
		011	y1
		012	y2
		:	:
		019	y9
		020	y(1)
		:	:
		028	y(9)
		029	K1(1)
		:	:
		037	K1(9)
		038	K2(1)
		:	:
		046	K2(9)
		047	K3(1)
		:	:
		055	K3(9)
		056	K4(1)
		:	:
		064	K4(9)
		065	K5(1)
		:	:
		073	K5(9)
		074	f(1)
		:	:
		082	f(9)

# Register Contents (Continued)

	Program Name	Register	Contents
Number Theory	Phi	000	n
		001	used
		002	$\phi(n)$
		003	used
	d	000	n
		001	d(n)
		002	used
		003	used
		004	used
	Sig	000	n
		001	$\sigma(n)$
		002	used
		003	used
		004	used
	Congruence	000	A and M
		001	a
		002	b
		003	m
		004	used
		005	x
		006	used
		007	used
		008	used
	Rational Approximation	000	used
		001	allowable error
		002	used
		003	A
		004	used
		005	used
		006	B
		007	used
		008	used



	Program Name	Register	Contents
Coordinate Transforms	Coordinate Transforms	000	x
		001	y
		002	z
		003	r
		004	theta
		005	phi
		006	rho
Analytic Geometry	Conic Sections	000	A
		001	H
		002	B
		003	G
		004	F
		005	C
		006	D
		007	J
		008	I
		009	K
		010	Type
		011	Omega
		012	used
	:	:	:
		028	used
	Quadric Surfaces	000	used
		:	:
		062	used
	Nonlinear Systems	000	order n
		001	$x_1$
		:	:
		008	$x_8$
		009	used
		:	:
		$14 + n^2 + 4n$	used
		111	used
	:	:	:
		114	used

## Flags Used

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The programs in the Mathematics library use one or more flags. If you write a program that uses a flag from the list below and then run a program in the library, the status of the flag may change.

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### List of Flags

The following table lists the programs in the Mathematics library and the flags used by each one.

Program Name	Flag 15	Flag 16
Complex Functions	x	x
Cubic Splines	x	
Exact Polynomials	x	
Gamma Function	x	x
Gauss Quadrature	x	
Matrix Product	x	
Inversion/Linear Systems	x	
Tridiagonal Systems	x	
Eigenvalues	x	
Polynomial Product	x	
Q-D Method	x	x
Bairstow Method	x	x
Bisection Method	x	
Newton's Method	x	
Runge-Kutta	x	x
Number Theory	x	x
Coordinate Transforms	x	
Conic Sections	x	
Quadric Surfaces	x	
Nonlinear Systems	x	

# Appendix B: Service and Warranty Information

This appendix describes the service provided by Texas Instruments and the warranty for the cartridge.

Table of Contents	Service Information	B-2
	One-Year Limited Warranty	B-4

## Service Information

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If you experience a problem with your cartridge, please call or write Consumer Relations to discuss the problem.

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### For Service and General Information

If you have questions about service or the general use of the cartridge, please call Consumer Relations **toll-free** within the United States at:

**1-800-TI CARES (842-2737).**

From outside the United States, call 1-806-741-4800. (We cannot accept collect calls at this number.)

You may also write to the following address:

Texas Instruments Incorporated  
Consumer Relations  
P.O. Box 53  
Lubbock, Texas 79408

Please contact Consumer Relations:

- Before returning the cartridge for service.
- For general information about using the cartridge.

### For Technical Information

If you have technical questions about the operation of the product or programming applications, call 1-806-741-2663. We regret that we cannot accept collect calls at this number. As an alternative, you can write Consumer Relations at the address given above.

### Express Service

Texas Instruments offers an express service option for fast return delivery. Please call Consumer Relations at 1-800-TI CARES (842-2737) for information.

### Calculator Accessories

If you are unable to purchase calculator accessories (such as carrying cases or adapters) from your local dealer, you may order them from Texas Instruments. Please call Consumer Relations at 1-800-TI CARES (842-2737) for information.

# One-Year Limited Warranty

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This Texas Instruments software cartridge warranty extends to the original consumer purchaser of the product.

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Warranty Duration	This cartridge is warranted to the original consumer purchaser for a period of one (1) year from the original purchase date.
Warranty Coverage	This cartridge is warranted against defective materials and construction. This warranty covers the electronic and case components of the software cartridge. These components include all semiconductor chips and devices, plastics, boards, wiring, and all other hardware contained in this cartridge ("the Hardware"). This limited warranty does not extend to the programs contained in the cartridge and the accompanying book materials ("the Programs"). <b>The warranty is void if the cartridge has been damaged by accident or unreasonable use, neglect, improper service, or other causes not arising out of defects in materials or construction.</b>
Warranty Disclaimers	<b>Any implied warranties arising out of this sale, including but not limited to the implied warranties of merchantability and fitness for a particular purpose, are limited in duration to the above one-year period. Texas Instruments shall not be liable for loss of use of the cartridge or other incidental or consequential costs, expenses, or damages incurred by the consumer or any other user.</b>  Some states do not allow the exclusion or limitations of implied warranties or consequential damages, so the above limitations or exclusions may not apply to you.
Legal Remedies	This warranty gives you specific legal rights, and you may also have other rights that vary from state to state.
Warranty Performance	During the above one-year warranty period, your TI cartridge will be either repaired or replaced with a reconditioned comparable model (at TI's option) when the cartridge is returned, postage prepaid, to a Texas Instruments Service Facility. The repaired or replacement cartridge will be in warranty for the remainder of the original warranty period or for six months, whichever is longer. Other than the postage requirement, no charge will be made for such repair or replacement. Texas Instruments strongly recommends that you insure the product for value prior to mailing.